Titre: Title:	Fast Fashion: Why Firms Incinerate Deadstock, and Public Policies
Auteur: Author:	Pedro Cybis
Date:	2022
Type:	Mémoire ou thèse / Dissertation or Thesis
Référence: Citation:	Cybis, P. (2022). Fast Fashion: Why Firms Incinerate Deadstock, and Public Policies [Mémoire de maîtrise, Polytechnique Montréal]. PolyPublie. https://publications.polymtl.ca/10529/

URL de PolyPublie: PolyPublie URL:	https://publications.polymtl.ca/10529/
Directeurs de recherche: Advisors:	Sophie Bernard
Programme: Program:	Maîtrise recherche en génie industriel

POLYTECHNIQUE MONTRÉAL
affiliée à l'Université de Montréal

Fast Fashion: Why Firms Incinerate Deadstock, and Public Policies

PEDRO CYBIS
Département de mathématiques et de génie industriel

Mémoire présenté en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*
Génie industriel

Août 2022

POLYTECHNIQUE MONTRÉAL
affiliée à l'Université de Montréal

Ce mémoire intitulé :

Fast Fashion: Why Firms Incinerate Deadstock, and Public Policies

présenté par **Pedro CYBIS**
en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*
a été dûment accepté par le jury d'examen constitué de :

Marcelin JOANIS, président
Sophie BERNARD, membre et directrice de recherche
Hassan BENCHEKROUN, membre

DEDICATION

À la transition …

ACKNOWLEDGEMENTS

I would first like to thank the people who contributed to the different steps of my master's degree and to the success of my efforts.

Thanks to my director, Sophie Bernard, who was able to guide me and provide me with valuable insights.

I would also like to thank the team at Polytechnique, who offered me much needed support to persevere in my studies despite the pandemic context.

I wish to acknowledge the supportive role played by my personal entourage: my parents, Walter and Rose, who took the time to read, criticize and encourage me by generously sharing their experiences, even when I was unsure of my success; and Myriam, who helped me get into action when I lacked focus.

To my friends, Fannie, Gabriel, Pascale, Brunno, Louise and all in Montréal and Paris: your encouragement, advice and approval made all the difference.

Annie and Carlos, thank you for giving me an opportunity to enter the industry that my dissertation discusses. This addition to my background and your insights have been thought-provoking.

I would also like to thank financial support for my project from the Réseau de recherche en économie circulaire du Québec through their axis "Léviers politiques" and the Smart Prosperity Institute through its Greening Growth Partnership, which is supported by a Social Sciences and Humanities Research Council of Canada Partnership Grant (no. 895-2017-1018), as well as by Environment and Climate Change Canada's Economics and Environmental Policy Research Network (EEPRN). Your grants allowed me to focus on my project and were a crucial factor in its success.

Finally, to the people I met during scientific events and who offered their constructive comments, I thank you. I have a special thought for the organizing committee and participants of the SCSE 2022 annual conference, who helped to take the thesis paper a step further.

RÉSUMÉ

La mode rapide augmente la consommation modiale et génère un besoin accru de nouveaux produits à chaque saison. Les tendances s'estompent rapidement, alors les produits restants perdent leur valeur et occupent un espace important dans les magasins. La destruction des stocks invendus contribue à l'impact environnemental déjà important de l'industrie de la mode. Dans un contexte de fast fashion, les détaillants et les marques commandent des quantités supérieures à la demande réelle, ce qui génère des stocks d'invendus. Ce stock en excédant doit être géré, mais il existe actuellement peu d'alternatives durables aux sites d'enfouissement et aux centres d'incinération.

La pensée fondée cycle de vie établit la pertinence d'aborder ces enjeux d'élimination en examinant la manière dont les produits sont conçus, fabriqués et distribués. La première partie de cette thèse est une vue d'ensemble de la chaîne d'approvisionnement conventionnelle d'un vêtement en suivant les flux de coton et de polyester depuis le stade de matière première jusqu'aux vêtements usagés en post-consommation. Elle met en évidence les principales activités générant des externalités négatives dans l'industrie de la mode et la façon dont le contexte mondialisé de l'industrie crée des pays consommateurs dépendants des importations pour leur consommation de vêtements tout en étant mal équipés pour gérer les résidus textiles. L'étude montre également comment la réutilisation et le recyclage sont des stratégies majeures pour une transition durable, mais fortement limitées par la saturation des marchés de seconde main et le manque de pratiques fiables de gestion des déchets textiles.

La deuxième partie de cette thèse est un article qui développe un modèle d'optimisation proposant une fonction de demande incluant le désir de nouveauté des consommateurs et l'élasticité croisée des vêtements considérés à la mode en début de saison et du même produit en solde en fin de tendance. Dans un premier temps, un détaillant résout un problème de production tout en considérant la présence de politiques publiques : une taxe sur l'élimination des stocks invendus et un tarif de responsabilité élargie du producteur. Ensuite, les quantités mises sur le marché dans cet équilibre sont comparées à un optimum social trouvé par un planificateur social maximisant la fonction de bien-être social. L'objectif est de mettre en évidence l'effet des incitatifs économiques sur la prise de décision du détaillant concernant les inventaires. En tant que coûts supplémentaires à la production, il y a une baisse attendue de la quantité totale produite. Ensuite, une analyse statique comparative est effectuée pour les paramètres. Notamment, la volonté des consommateurs de payer pour la nouveauté fait pression sur les détaillants pour qu'ils mettent sur le marché des collections à la mode à

chaque saison, ce qui pousse à une augmentation des prix pour soutenir cette production.

Comme le montrent à la fois la revue de littérature et le modèle, les politiques publiques sont essentielles pour soutenir le développement et l'adoption de solutions pour limiter l'impact des externalités de l'industrie. Étant donné que les stocks invendus sont observés et attendus des entreprises, il est nécessaire que les décideurs politiques réduisent les incitations et les mécanismes existants conduisant à la création et destruction des stocks en excédant.

ABSTRACT

Fast fashion increases the overall consumption and need for new products each season. With trends rapidly fading away, remaining products lose their value and occupy precious space in stores. The destruction of deadstock contributes to the already significant negative externalities from the fashion industry. In a fast fashion context, retailers and brands order larger quantities than the actual demand, which generates unsold inventory. This excess stock needs to be managed, but, currently, there are little sustainable alternatives to landfills and incineration centers.

A lifecycle mindset establishes the relevance of addressing these disposal issues by looking into how products are designed, manufactured, and distributed. The first part of this thesis is an overview of an apparel's conventional supply chain following commodity flows of cotton and polyester from their raw material stage to worn clothes. It highlights the fashion industry main activites generating externalities and how the globalized context of the industry creates consumer countries dependent on imports for their fashion consumption while ill-equipped to manage textile waste. The review also exposes how reuse and recycling are major strategies for a sustainable transition, but strongly limited by the saturation of second-hand markets and the lack of reliable textile waste management practices.

The second part of this thesis is an article which develops an optimization model that proposes a demand function including consumers desire for newness and the cross-elasticity of high fashion content clothing at the start of the season and the same product on sale at the end of the trend. At first a retailer solves a production problem with the presence of public policies: a tax on disposal of deadstock and an extended producer responsibility fee. Then, the quantities brought to market in this equilibrium are compared to a social optimum found by a social planner maximizing the social welfare function. The goal is to highlight the effect of economic incentives on the retailer's decision-making regarding inventory. As additional costs to production, there is an expected decline of the total quantity produced. Afterwards, a comparative static analysis is carried out for the parameters. Notably, consumers' willingness to pay for novelty puts pressure on retailers to bring to market fashionable collections each season, which pushes a price increase to support their production.

As shown both in the literature review and the model, public policies are critical to support the development and adoption of better practices that limit the industry externalities. As unsold inventory is expected and observed by firms, there is a need for policymakers to reduce incentives and existing mechanisms leading to deadstock destruction.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ACRONYMS

EPR	Extended Producer Responsibility
GHG	Greenhouse Gas
PET	Polyethylene terephthalate

LIST OF APPENDICES

CHAPTER 1 INTRODUCTION

Apparel is the third-largest manufacturing industry [1]. Our linear way of producing, using, and disposing of clothing generates significant environmental and social impacts that require a systemic change in order to transition towards a sustainable industry [2,3]. The destruction of unsold inventory by brands and retailers intensifies this already heavily impactful industry. The adoption of this strategy is increasing due to the world's growing consumption and the fast fashion phenomenon. In particular, this dissertation explores how retailers contribute to these issues by generating surplus on purpose and, for a lack of alternative disposal methods, destroying them in the end of a season. An original optimization model is developed that takes into account consumers desire for newness, the cross-elasticity between products and introduces economic incentives from public policies. The analysis of the equilibria found gives important insight on the effects of the implementation of a tax on disposal of deadstock and an extended producer responsiblity fee.

1.1 Concepts

1.1.1 Fashion Industry

The fashion industry is the application for over 70% of textiles globally [4], with polyester and cotton as the main materials in the market [5, 6]. From this raw material extraction and fiber manufacturing, the linear production follows with yarn spinning dyeing and treatment for finishing, and processed into fabrics before being sent to clothing manufactures. Then, the end product (clothing/apparel/garment) is distributed to consumers through retailers worldwide.

Industry Size, Growth and Global Context

Apparel is an ever increasing consumer industry expected to generate trillions by 2030 and employing millions [4, 7]. This growth follows the world‘s population increase along with their purchasing power.

Supply chains in the industry have been adapted by stakeholders to ensure the most significant market share for brands, which makes the industry market driven. Like other industries, the apparel must deal with the integration of its stakeholders to increase benefits from collaboration [8]. Particularly, brands and retailers have taken further control of logistics and operations [8–10]. It can be argued that it transformed the competition between companies into a competition between supply chains [9]. Paradoxically, parts of the supply chain, like yarn spinning, fabric production and garment manufacturing, remain highly fragmented.

The industry is deeply globalized, and yet, Asia is still the major producer and exporter region with China and Bangladesh at the forefront. However, the industrial poles shift in the region, and inside countries, as communities move from manufacturing to other industries, notably to services [11]. This change happens to improve their social welfare as the fashion industry has a significant environmental and social impact.

Fast Fashion

The fast fashion phenomenon is far from new, and it has shaped businesses in the industry by making them adopt models that required a fast and flexible supply chain. Clothing characterized as fast fashion have a high fashion content, are influenced by rapid trend cycles and have fast lead times [12, 13].

As apparel is designed for function and aesthetics, it is dependent on seasonality due to weather, but mostly due to which trend is fashionable at the moment [14]. Shifts in culture and popularity influence consumers' demand, which creates new fashion trends and a desire for newness [15]. This is related to the main effect of fast fashion, which has been to significantly increase the number of released collections annually [14].

The fast fashion phenomenon and its impact on the industry are further discussed on Chapter 2 Literature Review.

Externalities

Negative externalities that translate into social and environmental impacts arise from activities along the fashion industry, notably during textiles and apparel manufacturing [3,7,16,17]. Externalities can be positive or negative and are an undesired effect of market failures to transfer real costs to firms [18]. This concept is used to estimate a monetary value for an impact not taken into account by the industry, such as environmental damages and social conditions.

The environmental impacts along the supply chain are mostly a contribution to climate change through greenhouse gas (GHG) emissions and an endangerment of ecosystems quality due to a heavy water footprint, land use, intensive use of chemicals, and plastic pollution from waste generation [16, 19, 20]. Indeed, the apparel industry is the cause of $7,2\%$ of the annual global GHG emissions [21], mostly due to a reliance on fossil fuels as energy source. For water use, the industry is responsible for $17\% - 20\%$ of industrial water pollution due to dyeing and finishing treatments [22]. Then, the industry generates significant volumes of waste through the lifecycle of a garment. It is 35% of input materials that are lost during production, which become post-industrial and pre-consumer waste. Unsold inventory in need of disposal enters in the pre-consumer waste category, because it is clothing that was never worn. After use, post-consumer textile waste has limited sustainable alternatives as the majority is sent to landfills worldwide [16].

The globalized context of the industry establishes an imbalance on who is affected by this environmental impact. It leads to an injustice where it is the communities that manufacture that suffer its damage [17]. In addition, due to the agricultural sector needed for cotton and lax workers' rights where production takes place, there are many social impacts to the fashion industry [1]. Notably, other than meager wages and health hazards, labor practices can be as poor as to create environments of modern slavery [7].

Due to time and cost constraints, fast fashion brands and retailers put additional pressure on their supply chains, which exacerbates these externalities.

1.1.2 Public Policies

The implementation of public policies can put regulations and economic incentives in place to either inhibit or compensate for negative externalities. Kelderman [23] census existing environmental policies applied to the fashion industry worldwide. Notably, North American and European governments are at the forefront of regulations concerning the fashion industry.

Tax on Disposal

An externality tax can be used to adjust the prices to the real cost including negative externalities [18]. Applied to the disposal of unsold inventory, a tax creates a bonus-malus scheme [23] were retailers that destroy their unsold inventory face higher costs than the ones avoiding this situation.

Extended Producer Responsibility

A mandatory extended producer responsibility (EPR) system introduces legal obligations to retailers to modify their operations and strategies in order to ensure a more sustainable disposal of their products after the use phase [23].

Similar to a tax, an EPR program introduces a fee that aims to make a producer internalise negative externalities. Applied to the disposal of excess stock situation, an EPR fee forces the retailer to cover for the environmental damage of production and end-of-life treatment.

Additionally, an EPR helps support and develop collection and sorting operations for textile waste. It is critical as their current limitations are significant barriers to the development of sustainable alternatives of disposal.

1.2 Elements of the Problem

It was infamously reported in 2018 that the luxury brand Burberry burned high quantities of inventory of accessories and clothing to preserve their elite image with the defense of recovering energy [3]. This scandal and the effects of fast fashion on the United Kingdom led to a report with recommendations from the UK Parliament's Environmental Audit Committee on the issues of the fashion industry. This example is relevant for the Canadian context as our industry is similarly based on fast consumption and imports.

There is a "symbiotic relationship between fashion firms and consumers" [2] that blurs the burden of responsibility for the intense environmental impact of garments. Thanks to fast fashion, more people have access to fashionable products for an affordable price, and people's lifestyles have increased clothing consumption since the 1980s [24]. This improves welfare as it fulfills the consumers' need for novelty more easily. Indeed, consumers have a general need for newness that is cultivated by the trend cycles and which is distinct from the essential need to dress [15, 25]. The downside of fast fashion is that garments are more quickly discarded and replaced, which creates a waste generation problem at the same time as new garments are needed. The supply chain's agility or responsiveness became essential with the dominance of fast fashion products, but it "increases unsold commodities, return rates, packaging materials and waste" [26] which intensifies the overall impact.

Furthermore, clothing and accessories are subject to a short life cycle, impulsive buying from customers, high demand uncertainty and obstacles to forecasting [27] . It increases the market volatility as well because of lower quality products [3]. At the end of a selling season and when the trend passes, garments have a significantly smaller value. This devaluation contributes to the problematic behavior of destroying unsold inventory [3, 28]. The same goes for returns that are not integrated into a reverse logistics channel capable of keeping the products value. It happens in luxury market segments but also in fast fashion.

The industry leaders are well-positioned compared to the 40% of firms that have done little to nothing in terms of sustainable initiatives [7]. Thanks to pressure from awareness campaigns and to ample resources, they have been able to adopt more environmentally friendly and ethical strategies than other businesses. Also, the communications on the implemented strategies are setting up the groundwork for a transition [10]. However, small and medium businesses form most of the industry's firms. Those find themselves with incentives to optimize profits and minimize costs regardless of the critical environmental and social consequences.

With the pandemic, the harmful impacts worsened as orders were cancelled early on [29], and a social cost was added to its economic cost [30].

Retailers and brands would turn to incineration and landfills as strategies of disposal for their unsold products because it can be economically more favorable to them. However, landfills or incineration centers are the lowest strategies in the waste disposal hierarchy [31, 32]. Instead, retailers and brands should invest on reverse logistics channels capable of preserving at least part of the product's value. Secondhand markets are the main closed-loop channel for garments in Canada, but textile-to-textile recycling options are in small scale or still in early development stage. Governments are also reinforcing their commitments to the need for regulations designed to inhibit the apparel industry's negative impact by discouraging these destructive strategies. Considering all this, how can public policies influence the environmental and social impact of the apparel industry?

1.3 Research Objectives

The goal of this research project is to explore the effect of public policies on retailers' practice of destroying their unsold inventory, with a focus on consumers desire for newness. More specifically, the sub-objectives are to:

1. Identify the mechanisms promoting the destruction of deadstock and relevant public policies with economic incentives;
2. Develop an optimization model of the fashion industry including public policies and additional externalities;
3. Estimate the effect of those public policies on quantities brought to market by a retailer and compare them with socially optimal quantities.

The suggested methods to achieve the sub-objectives are two-fold. For sub-objective 1, a literature review was conducted to best describe the fast fashion industry, its environmental impact and existing public policies applied to the context. Then, for sub-objective 2, an original optimization model was developed, based on the findings of sub-objective 1, to include two economic incentives from public policies. Additionally, the model aims to internalise negative externalities such as the environmental impact occurring during production. Finally, a static comparative analysis was done to achieve sub-objective 3.

1.4 Outline

Following this short introduction, this research project follows with a literature review, a section explaining the general approach, and an article that develops the method and presents its results. Then, a discussion on those findings, stating the strenghts, limits and perspectives for future research is followed by a conclusion.

The body of work on Chapter 2 is a literature review, but also further examines sustainability aspects of the fashion industry. It was first published as a working paper with the support of the Smart Prosperity Institute in October 2021.

The results of the article on Chapter 4 were presented on the 61e annual congress of the Société canadienne de science économique held from the 11th to 13th May 2022. It introduces the original optimization model and presents its results and findings.

The appendixes that further detail the steps of the optimization operations can be found at the end of the document. Appendix A goes through the retailer's production problem, while Appendix B demonstrate the steps for the social optimum.

CHAPTER 2 LITERATURE REVIEW

Pedro Cybis, Sophie Bernard. (2021). Fast Fashion: Why Firms Incinerate Deadstock, and Public Policies. Text published in the Clean Economy Working Paper Series by the Smart Prosperity Institute in October 2021.

2.1 Fast Fashion

A positive take on fast fashion is that it brings new, and trendy looks to consumers unable to afford luxury items. Brands and retailers strive to fulfill this consumers' desire for newness at an affordable price and in time for the rapid change of trends. In turn, it impedes retailers from forecasting the demand for new products, which compromises the accuracy of quantities ordered to manufacturers. Hence, fast fashion brands and retailers have established supply chains that are fast and agile enough to meet consumer's expectations. Paradoxically, retailers also require a more standardized supply chain to improve product variety by offering basic articles in their apparel groups [12, 14]. Such variety in apparel articles and the capability to develop products quickly and bring them to market increases a firm's market share and competitiveness [12]. It increases consumption and heightens the apparel industry's impact on social and environmental dimensions.

Additionally, fast fashion trends follow seasonality, going up to five seasons per year for some retailers [14]. Each seasonal collection has multiple apparel articles resulting in retailers having to introduce new products on store floors every week or two weeks [14]. Once the trend fades, a fast fashion garment loses value because its high fashion content becomes outdated. Advertisement and social media contribute to this overconsumption by "selling a message of satisfaction after purchase" [33]. With the fast fashion segment leading growth, consumers want to renew their wardrobe and stay on trend. They spent up to 60% more on clothing between 2000 and 2014, while keeping the garments for only half the time [34]. With Canada being a top 10 importer [35], it can be expected that its population displays a similar rate of consumption.

McNeil and Moore (2015) focus on what influences the compromise that customers must make between their sustainability awareness and a fashion consumption that fulfills their need for identity construction. The authors highlight that fast fashion consumers have a frequent purchase rate, are motivated by enhancing their image and try to fit in with peers. There is hope for sustainable purchases, and customers care for a brand's responsibility, but it is yet a concern to be seen in real consumer behavior because of prices [25] . This rapid obsolescence is one of the reasons apparel has a short lifecycle, but it is also caused by low quality products resulting from the focus on speed in the supply chain compromising tests [3].

Furthermore, retail stores find themselves intertwined with customers exhibiting various behavioral patterns. When determining their inventory and pricing policy, retailers have to consider strategic consumers' purchase decisions. Cachon and Swinney (2009) modelized the equilibrium between the retailer's interest and strategic consumers, expecting prices to go down during the selling season. Indeed, markdown prices are part of dynamic price policies and retailers have far more strong incentives to choose this strategy than a static fixed price [36].

In the last decade, retailers have consolidated around larger brands and started private labels [14]. Because retailers are the closest to consumers and their demand, this gives them an advantage in designing trendy products, which become enhanced in their value [13]. Additionally, retailers started controlling manufactures' production in a vertical integration way, which reduces their costs and the time needed. These two advantages are particularly present in fast fashion retailers and are heightened in the presence of highly strategic consumers as both reduce incentives to delay a purchase [13]. Hence, successful fast fashion retailers would invest resources into ensuring the speed of their supply chain and that their products have a high fashion content. However, following the latest trends this way and with such speed exacerbates the obsolescence of these garments.

In 2020, the retail part of Canada's fashion industry's worth was close to 2 billion CAD [37], whereas the whole clothing product manufacturing was 1,2 billion CAD [38]. It reflects well how retailers have become major decision-makers in countries where the consumption of apparel is dependent on import.

2.2 Newsvendor Problem

2.2.1 Stochastic Demand and Risk Preference

With the typically short lifecycle and demand uncertainty, inventory management for fast fashion retailers exemplifies the newsvendor problem, which forces a firm to make a decision on quantity and prices without precise knowledge of demand for the product. The uncertainty in the demand forecast provoked by fast trend cycles and seasonality can be represented by a stochastic model [39]. Hence, in the fast fashion context, the consumer demand becomes prospective. The newsvendor problem assumes no restrictions and no lead time for suppliers as well as a risk-neutral buyer [39] . It is not the case for the apparel industry, where orders must be put months in advance and buyers are not necessarily risk-neutral. Indeed, brands and retailers have variable risk preferences that need to be considered. The stochastic demand and the risk preference from decision-makers ordering quantities affect the optimization of the production. Adhikari et al. (2020) revealed how the risk preference of retailers influences a cotton garment's overall supply chain performance.

2.2.2 Loss-Averse Retailer

If a retailer is considered loss-averse, their preference will be to avoid losing revenue over gaining the same amount. In this case, they will order less than the future demand in fear of having spent too much. However, ordering less could lead to missing sales and trying to avoid that scenario refers to a stockout-averse preference [40]. In this scenario, fearing the loss of sales and customer loyalty, retailers, and brands order larger quantities than the expected demand during the selling season [40]. For a fast fashion garment, this safety inventory works as a buffer, ensuring no customer is dissatisfied because they missed out on the newest trendy garment. After the selling season, however, the unsold inventory has very little salvage value because of the passing trend's obsolescence. These insights help understand the problems of generating unsold inventory that ultimately entails storage costs or disposal costs.

An important addition to the question of how to dispose of unsold inventory is dealing with returned articles. Reverse logistics enables channels to reintroduce these returned products for reselling or direct them for reuse in secondhand markets. However, the efforts to adopt such strategies are rarely given by decision-makers [41]. Like unsold inventory, returns also have little value after the selling season and are sent to landfills or incinerated. It is also the case for returns from online shopping [42]. Fast fashion has the effect of increasing the quantities of garments in these situations.

2.3 Incentives to Destroy Deadstock

Retailers and brands in different market segments of the fashion industry have varied kinds of incentives to destroy unsold inventory. Sought outcomes range from simply freeing stock space to protecting a brand's image or trying to benefit from tax credits given for energy recovery through incineration [28].

For a fast fashion firm, Napier and Sanguineti (2018) identify a cause of stock destruction as excess production related to miscalculations in ordered quantities. However, the link between the destruction of unsold inventory and its devaluation at the end of its selling season is ambiguous when taken into account that the disposal strategy is already planned and established during production. Furthermore, fast fashion brands and retailers have established supply chains that allow fast speed to market and inventory resupply. Two advantages that minimize errors due to uncertainties induced by rapid trends, which justifies the investments made on responsive supply chains. Hence, it can be argued that the ordered quantities are optimally calculated to exceed expected sales. In a loss-averse scenario, it is the anticipated strategy that a retailer would adopt to maximize their profits [40].

Interestingly, for luxury brands, stock destruction is an issue of brand image protection [28, 31]. Specifically, they try to avoid situations where their products are sold under large discounts as it compromises the brand's elite image. Destroying excess stock avoids this outcome and contributes to ensuring product scarcity [28].

In general, for players from all market segments, the costs of implementing and maintaining strategies remain an important barrier to systemic change [31]. Particularly, there is a need for reverse logistics, which are operations that allow brands and retailers to retrieve worn garments from consumers once the later seek to dispose of it. To incentivize firms to make sustainable commitments and take tangible actions, there is a critical need for access and accountability to their own end-of-life products.

2.4 Fashion Industry Environmental Impact

The textile and apparel industry has an environmental impact that contributes heavily to GHG emissions, water scarcity and pollution, and waste generation [19, 20, 43]. Fast fashion's intrinsic characteristics exacerbate these impacts by increasing the rate of production, consumption, and disposal, especially when retailers and brands destroy their unsold inventory. This becomes even more important considering the expected increase in purchasing power of emerging economies [34]. The "race to the bottom" for minimizing costs by localizing operations where working conditions and environmental regulations are weaker [11] also contributes to the impact of operations along the supply chain.

2.4.1 Carbon Footprint

In 2018, the consulting firm Quantis contributed to the understanding of the industry's global impact with their report "Measuring Fashion: Environmental Impact of the Global Apparel and Footwear Industries Study". Their lifecycle assessment of a garment highlights which phases of production are the most impactful and for which damage category from the IMPACT 2002+ method. They determined that the primary contributors are the energy sources based on fossil fuels that fabric manufacturers use [21]. The steps preceding a garment's confection are the phases requiring the most of these resources. Hence, fiber production, yarn spinning, fabric manufacturing, dyeing, and finishing treatments encompass nearly 80% of a garment's impact. Types of fibers add nuance to the impact of those production phases as they require different processes. For damages related to climate change, synthetic fibers intensify GHG emissions as, with over half the world's market shares [6], they are the most used materials, with polyester fabric dominating the market. Its manufacturing is concentrated in Asia, particularly in China, and by petrochemical companies that produce the PET needed for polyester [4]. The findings of Quantis are aligned with the "Environmental assessment of Swedish fashion consumption" done by Mistra Future Fashion that highlights the impact taking place during production phases as far more significant than use and disposal [19]. Additionally, the transport of materials and components worldwide is another leading contributor to GHG emissions because of the globalized and fragmented nature of the industry [11].

2.4.2 Ecosystems Quality

Another important damage category relevant to the industry is the degradation of the ecosystems quality. Dyeing and finishing treatments for fabrics are chemical-intensive processes, but needed for garments' aesthetics and functionality criteria. Fabrics have finishing treatments for specific attributes (e.g. softness, water repellant, flame retardant, etc.) and color dyes change rapidly depending on trends. These processes require a particularly intensive water use responsible for most of the water footprint of a garment be it made of cotton or polyester [19].

Furthermore, pesticides are posing a significant danger for biodiversity in regions with intensive cotton cultivation [44]. Cotton also has a particularly strong impact on ecosystems quality, as it requires vast stretches of land, amount of water and chemical products to produce the high volumes demanded by the textile industry.

2.4.3 Linear End-of-Life

The Ellen Macarthur Foundation published, in 2017, an essential report about sustainability in the fashion industry highlighting how its linearity generates important amounts of waste annually. One of the most striking conclusions of "A New Textiles Economy: Redesigning fashion's future" is how textiles' material flow has only 2% of input material that comes from recycled products originating in other industries and so few channels to recycle end-of-life garments into new pieces of clothing. Instead, while textile-to-textile recycling represents 1% of disposed textiles, 12% of used garments are downcycled in other industries, and 73%

end up in landfills or incinerated [16]. The rest of end-of-life textiles are lost either during production, collection or through washing. It becomes worrisome when waste hierarchy is taken into consideration. Waste prevention and treatments that preserve utility, or material value at the very least, are recommended over final disposal methods [32].

2.5 Apparel Supply Chain

2.5.1 View of Interests

Omnipresent, the apparel industry today is a good example of a globalized industry. Picking up steam during the Industrial Revolution, production started relocating to where costs were reduced, and profits maximized. Since then, this reality has continued to happen [8, 11], resulting in many stakeholders worldwide navigating diverse interests and barriers.

Fast fashion transformed the conventional supply chain into a business model that can answer challenges related to a product with a short lifecycle and demand uncertainty. Barnes and Lea- Greenwood (2006) summarize that a quick response supply chain answers the needs for fast fashion by allowing the supply chain to identify trends and adapt to them thanks to the flexibility given by a fast speed to market. However, they show that fast fashion became a concept on its own as it expands further than the supply chain characteristics and "is a completely consumer- driven process" [15]. Its responsiveness leads to a virtual vertical integration, where the decision-makers of each stage are coordinated if not owned by the same corporation [15].

Brands were conventionally the main decision-makers regarding production since they design, put orders in with manufacturers and decide on pricing strategy. It is still the case for the luxury market segment. Considering the newsvendor problem, quantities ordered by brands are prospective because the demand for the new product is stochastic. On their side, garment manufacturers and textile producers try their best to prepare for the brands' order each season. The result is a realized production quantity based on a future order that is already a projection of sales [3]. Even with the agility of a fast fashion supply chain, there are important uncertainties for the demand of each apparel collection. It becomes even more relevant as those have increased in numbers.

Nonetheless, a shift took place and retailers became the industry's new primary decision-makers when private labels from stores increased in numbers during the early 2000's [15]. To adapt products' design accordingly and on time, decision-makers need to access information on fashion trends [13]. Retail buyers have the most advantageous position as they are closer to the data and feedback from consumers' purchase behaviors. This closeness with the demand reduces uncertainties for retailers. Hence, just like brands, retailers could offer private labels with high fashion content that meets trends and demands of consumers, but cheaper than a luxury market price. In addition to trends, there is time pressure due to the competitive environment, increasingly demanding customers and strategic behavior. Today's industry highly values the fastest lead times, while keeping low costs is a challenge that motivates new supply chain structures. More than quick response supply chains that adopt lean and agile strategies, it can be argued that the fashion industry developed its own distinct concept due to the market characteristics [8, 24].

However, fast lead times of a fast fashion supply chain puts pressure on quality control processes [3], which results in a low-quality product. The other part of the competition for low-quality garments is their low-price, which appeals to a broader range of consumers. Nowadays, with the digitalization of commerce, being competitive based on pricing is even more vital. The factor of fast lead times would also indicate that order sizes are optimized due to the retailer's closeness with the consumer demand, which should minimize waste along the supply chain. It is an argument that is made in favor of a fast fashion business model with short lead times [13].

Still, trendier garments quickly reach an end of useful life and become post-consumer waste, especially in the context of fast fashion. Due to the linearity of fast fashion products' life cycle, waste prevention during production does not reduce the end-of-life waste related to a rapid consumption rate. Hence, it becomes crucial to look at how garments are manufactured, consumed and at what are post-consumer waste treatments alternative to landfills and incineration centers. Which are the primary methods of disposing of worn garments.

2.5.2 Mapping the Linear Supply Chain

The stakeholders in the apparel industry have different interests, but they would all profit from further collaborating in reducing waste and closing the loop on material flows. A start has been seen since the 2000s, when the industry saw the adoption of lean, agile and just-in-time strategies to improve speed to market and the flexibility of orders [8]. However, the supply chain remains vastly linear, and it is relevant to understand how this conventional linear model works.

Method

The apparel industry is significantly globalized, with commodities flowing between countries worldwide [11]. Tracking where the materials originate in the supply chain and where they are sent from one production stage to another is a useful tool to understand and situate garments' lifecycle. From a take-make-use supply chain, the sequence of production and distribution stages starts with raw material extraction, fiber production, fabric production, garment manufacturing, and ends with the retailer [45]. Then, the use phase and the end-of-life treatments were added to consider a garment's environmental impact for its whole lifecycle. The Harmonized System Classification (HS) was helpful to categorize commodities [46] as well as to determine the relevant commodities to follow. The International Trade Center (ITC) website (www.trademap.org) was also useful to determine which countries are the top exporters and importers. The ITC mainly uses data from the UN Comtrade database and Statistics Canada for Canadian imports and exports. The next step was to verify data availability. For a reliable portrait of the commodities flow, the most recent year suitable for reference is 2018. Hence, it was chosen as the reference year. The goal was to determine the main countries involved in order to confirm initial assumptions and identifying new leads for investigation.

From Fiber to Garment

Fibers

According to Textile Exchange, a nonprofit organization that coordinates the industry to promote preferred materials, synthetic fibers dominate the global market, followed by natural (cotton) and then man-made cellulosic fibers. For the year 2018, polyester alone represented 51,5% of global market shares, whereas cotton was 24,4%, and artificial fibers like man-made cellulosic (MMC) fibers were 6,2% [5]. The following year, their share of the global market increased slightly [6]. Figure 1 illustrates the distribution of these main fibers in the global market.

Other types of materials used for garments, like silk, wool, and down were not considered because each was less than 1% of the market share. Those materials and leather are considered noble materials. As such, they are more expensive and mostly used for high-end or luxury apparel. For an analysis of fast fashion garments, it is less relevant to track those materials' flow. Hence, only the three primary fibers (synthetic, natural, artificial) are initially considered. Respectively, polyester, cotton and viscose dominate each of the top fiber categories. Understanding the main fibers is relevant as their blend into fabrics is one of the main technical obstacles to textile recycling.

Polyester

Polyester, made from polyethylene terephthalate (PET), is the main synthetic fiber and represents by itself 51,5% of 2018's global fiber market [5]. PET is spun into threads by different processes (e.g. melt spinning) before being sold as yarns. Because of polymers needed for polyester, petrochemical companies are well positioned to produce the PET made into polyester yarns. It is a form of vertical integration, where manufacturers of raw materials are also the ones producing fibers and fabrics. Companies adopting this strategy are concentrated in Asia, particularly China [4].

Additionally, polyester has the potential to play a role in the industry's transition toward a circular economy. Recycled PET's (rPET) part of the input of raw materials for polyester increases each year and creates opportunities for an economically viable channel for recycled PET. Indeed, last-mile plastic collection projects reduce quantities of plastic bottles that reach oceans. The development and affordability of business models incorporating recycled fibers also lead to positive impacts for those companies. Still, polyester is made of fossil resources, and 75% of the raw material used is still virgin PET [5], which increases its GHG emissions and impact on nonrenewable resources. Closing the loop on end-ot-life PET with rPET is a way of reducing the pressure on the demand for fossil resources.

Man-Made Cellulosic Fibers

MMC fibers are manufactured from organic materials through processes of transforming cellulose into artificial fibers. Like synthetic fibers, it is a MMC fiber, however artificial fibers are made from organic materials. Hence, it has a powerful circularity potential as it can be made from other industries' waste like woodshedding. Lenzing is a good example of the sustainability issues that MMC fibers can address as they developed a fiber manufactured partly with waste cotton [47]. However, viscose still dominates artificial fibers by representing 79% of man-made cellulosic fibers' share on the global fiber market [6] and it has its own issues. Unfortunately, as viscose is mostly made from wood, and some can be made of FSC-certified wood, this material is often linked with deforestation and exploitation of protected rainforest. As with the case of synthetic fibers, artificial fibers are also predominantly manufactured and imported by Asian countries.

Cotton

Cotton has important characteristics that need addressing. Its cultivation occurs worldwide in warm climates, with China, USA and India being the largest producers of raw cotton [48]. However, Figure 1 shows that the USA is by far the primary exporter of raw cotton, and China is the leading importer. It indicates two interesting realizations. First, as cotton crops are cultivated globally, it is also produced with very different levels of technology and institutional incentives. The USA does not have a strong textile manufacturing industry as it once had, but Texas is still a cotton production powerhouse [11]. Hence, its cotton is exported to international markets where it competes with cotton from other exporters like Australia, Brazil and West African countries. Secondly, China's production is directed towards their own domestic market, and it still does not fulfill the demand for raw cotton. From this raw material form, it needs to be cleaned through ginning before being spun into yarns. The findings match the known assumption that the textile industry is still a leading economic driver in China even if the sector is changing [11].

Fabrics

Once raw materials and fibers are transformed into threads, it is spun into yarns to be sold as an intermediate product and transformed. These processes are crucial as it conditions the threads to work with the machines that weave or knit them into fabrics.

It is uncommon for fabric rolls to be made of purely one fiber type, as it is often blended with other materials to achieve functional criteria. Polyester, mostly, is often mixed with cotton or viscose to allow the textile to become more breathable. Whereas cotton is often mixed with elastane to add flexibility to the garment. The result is a fabrics market mostly composed of blended textiles.

Dyeing and finishing treatments can take place in different stages of production depending on the desired result. Techniques vary as some producers might dye fibers while spinning the threads into yarns, while others do it once the fabric is already woven or knit. The substances, such as hazardous chemicals, are essential for the end result and, thus, the value of the apparel and its performance during use. However, other than their impact on ecosystems and workers' health, there is also a consideration for how these substances can affect and compromise recycling processes.

Figure 1 shows how the leading exporters of yarns are also the Asian countries that either had a domestic production or imported the raw materials. It is interesting to highlight that cotton continues to flow towards China. Indeed, the fabrics market is very fragmented but remains regionally concentrated in Asia and particularly in China and India due to the "low labor cost and predominant apparel consumption" [4].

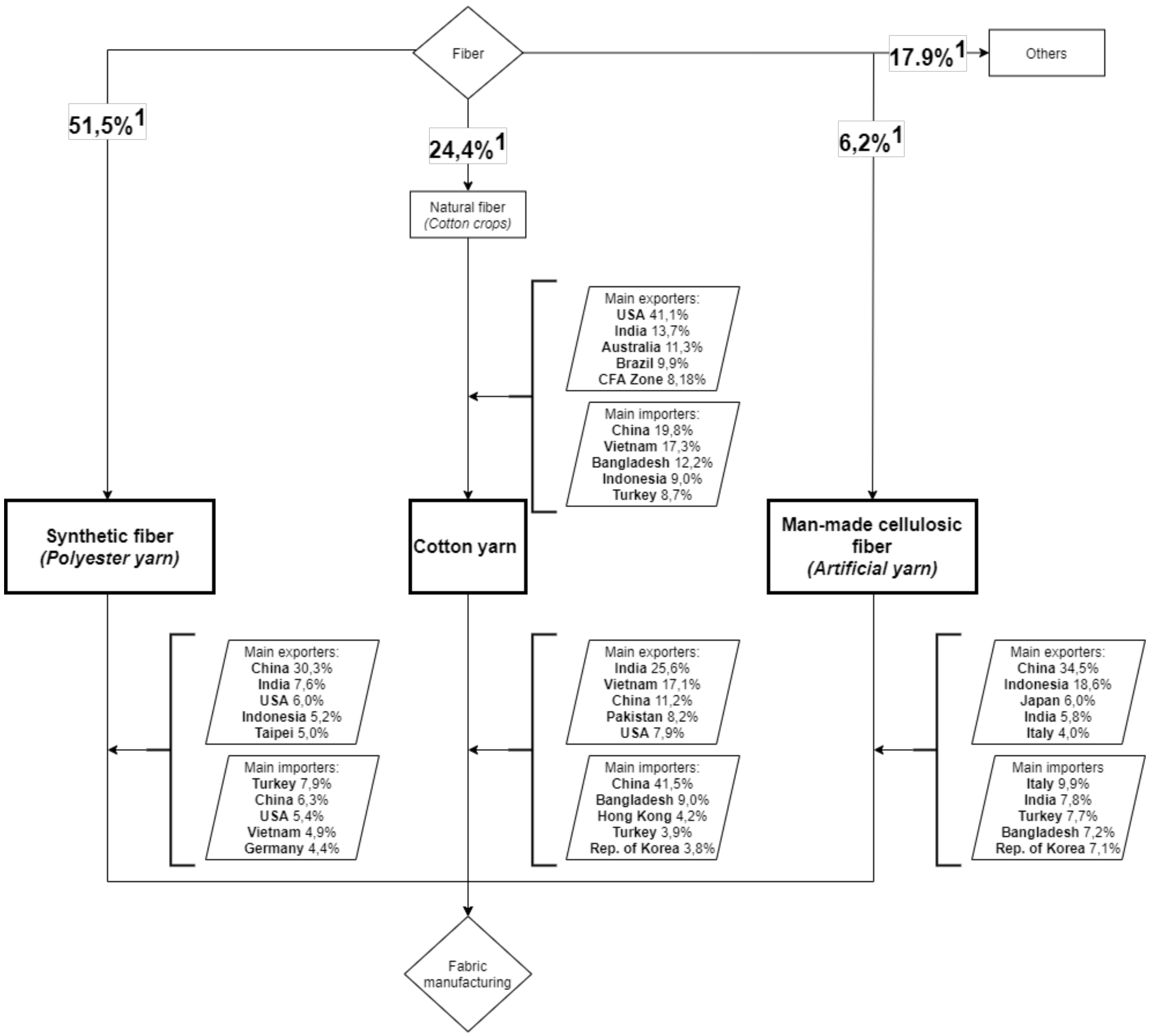

Figure 2.1 From fibers to the fabric.

Note: Trade flow data from International Trade Center, Trade Map - Canada, retrieved online in August 2020
Source: 1. Textile Exchange, 2019

To achieve fabrics with various looks and functionalities, different types of fibers can be blended. Often, cotton or polyester are mixed with small quantities of man-made cellulosic fibers to add an aspect of elasticity and comfort to the apparel. The HS Classification allows for tracking blends, but it still refers to the quantity of cotton or polyester in the yarn, fabric, or garment. Viscose and other fibers that are added for performance criteria become less trackable. Indeed, either with cotton or polyester at 85% and more, the codes' categories remain in those same two main types of fibers in the market. Figure 2 further explores them by looking at woven and knit processes. It is interesting to note that the global market value of woven cotton and polyester fabrics is thrice higher than the knitted ones.

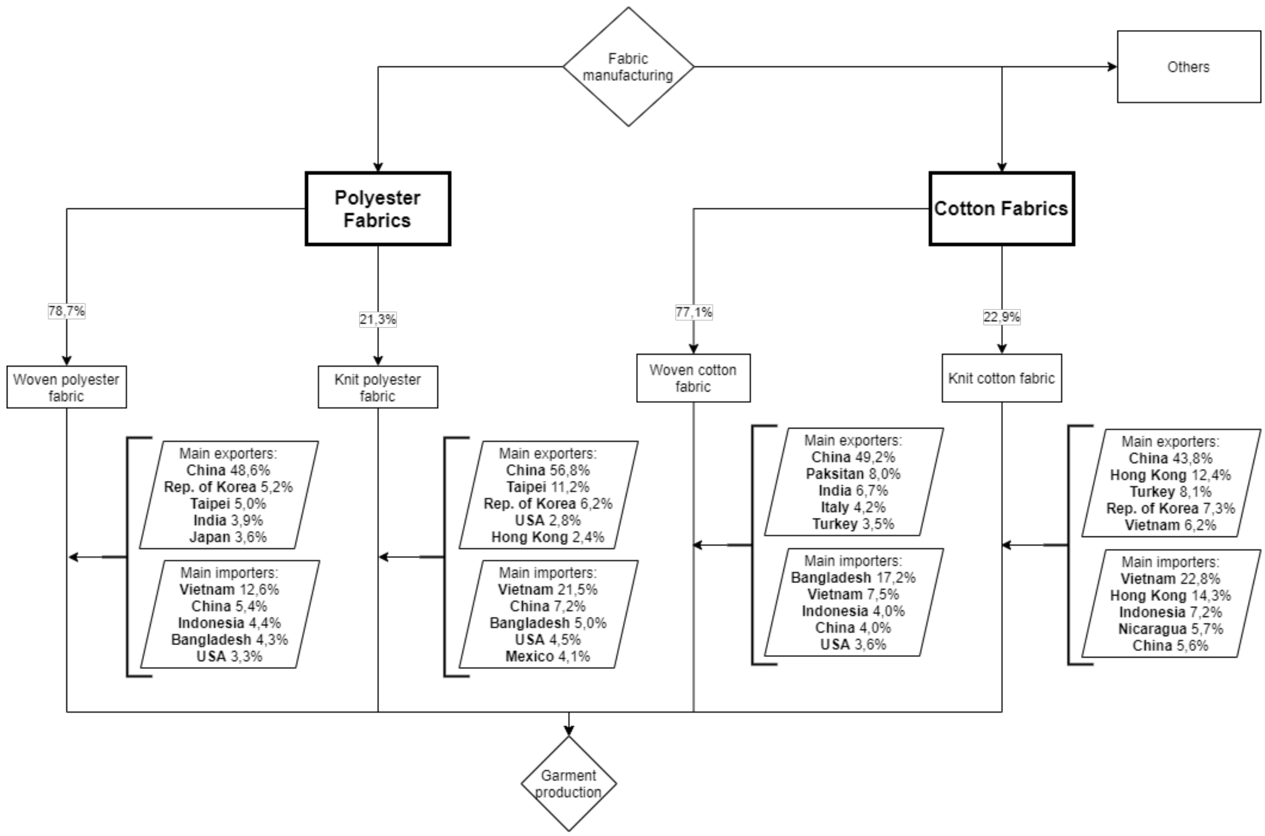

Figure 2.2 From woven and knit fabrics to garment manufacturing.

Note: Trade flow data from International Trade Center, Trade Map - Canada, retrieved online in August 2020

Garments

Exported Garments cross similar borders to the commodity flow of fabrics. Indeed, as Vietnam, Bangladesh and China import fabrics, these countries are the top exporters of garments in the global market. Figure 3 shows which countries represent the largest consumer market. As expected, garments originate in Asia and flow towards the USA, United Kingdom and Germany.

However, it is interesting and important to note the presence of Japan in the top five world importers. It highlights how Asian markets have an increasing appetite for fast fashion apparel. This is in line with the assumption that markets in developing countries will continue growing [7]. For the future of the industry, it is key to monitor the evolution of these developing markets and how they will face the environmental issues.

While yarn manufacturing and textile manufacturing are fragmented markets with a multitude of agents, revenue from clothing manufacturing is concentrated in a few multinational companies worldwide.

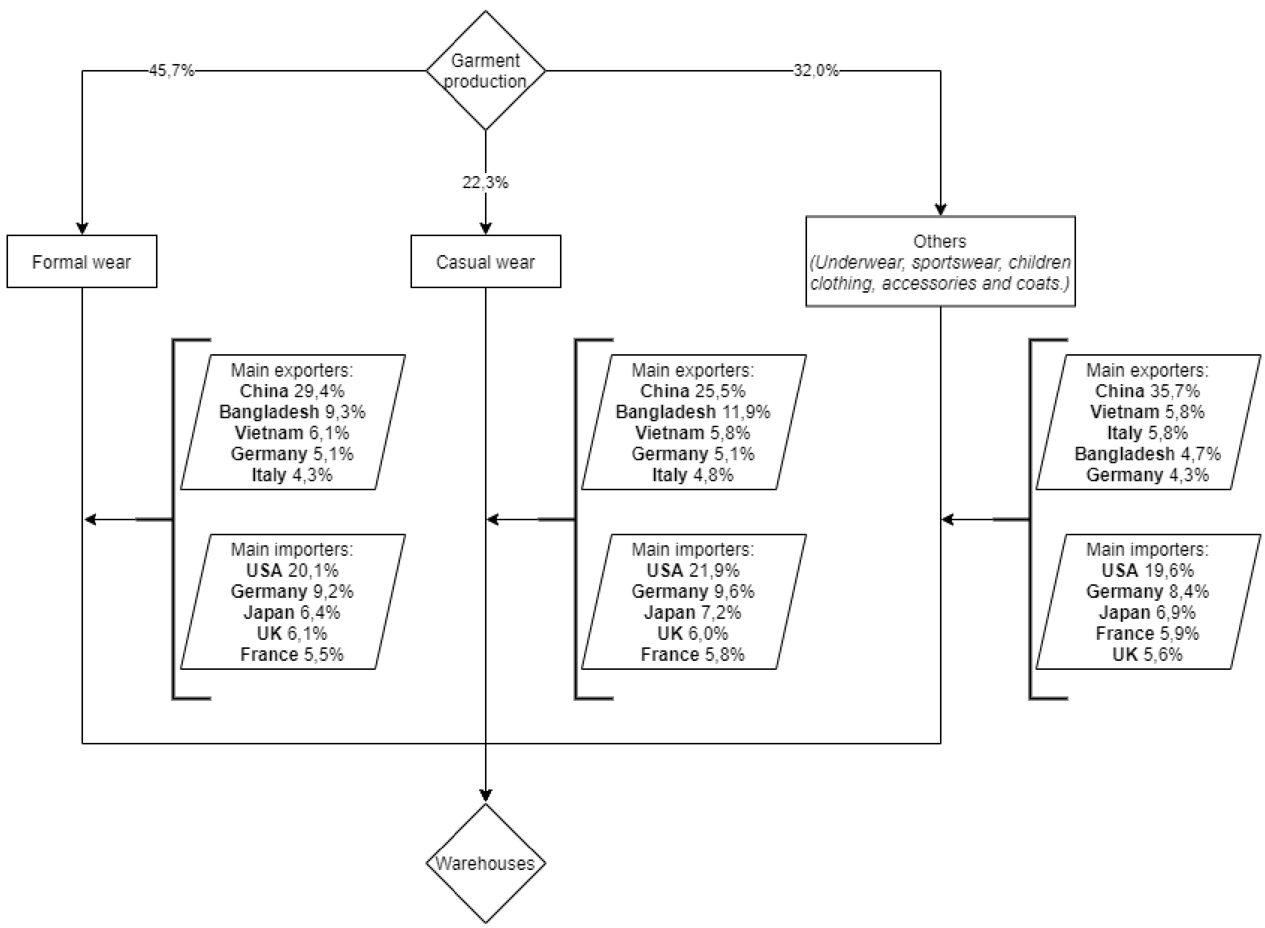

Figure 2.3 From garment manufacturing to stores.

Note: Trade flow data from International Trade Center, Trade Map - Canada, retrieved online in August 2020

End-of-Life

It was useful to track commodity flows to observe the origins and destinations of materials and their transformation to grasp the globalized nature of the industry. However, it becomes flawed when trying to analyze where the end-of-life treatment for post-consumer garments that became waste mostly takes place. For apparel, HS codes are limited to track worn garments destined to second-hand markets when identified as such, but it becomes useless for other disposal methods. End-of-life disposal alternatives like landfills and incineration are mostly local from curbside collection or regional operations including industrial waste. However, municipalities are rarely equipped to collect and sort textile waste appropriately. Recycling processes for textiles are still under development [47], and the volume of waste directed to such alternatives is limited [16]. Hence, we lose track of the volume of waste when using the commodities flow because worn garments do not cross international borders to be disposed of in these ways.

Reuse is the most present alternative to avoid these final disposal methods. Worn clothing sent to secondhand markets is an interesting but limited channel for retailers and brands looking for ways to dispose of unsold inventory and returns.

Indeed, second-hand market notable exception to the tracking flaw because donated garments can be tracked as they are exported. Reuse would be the preferred disposal type over other methods as it preserves the garment's function, material and energy used for production. From the main countries importing garments in Figure 3, the United States, United Kingdom and Germany are also the leading exporters of worn clothing in Figure 4.

However, regional markets for donated apparel are already saturated where the consumption of clothing is higher [3]. Indeed, based on the United Kingdom situation, an oversupply of donated and collected garments will devaluate the product in the global market, which could bankrupt the already challenging collection and sortation industry [3]. A low value for worn garments imposes barriers to make operations depending on it profitable (e.g. collection, sortation, recycling) as margins are small [41].

As donated apparel floods international markets, they end up directly in landfills around the world when quality and sanitation are too low or when they reach their end-of-life. Plastic leakage from synthetic fibers in mismanaged landfills can heavily contribute to microplastics entering ecosystems and water sources [49].

Unfortunately, important barriers are still present as collection and sortation of used garments face logistic and financial obstacles to closing the industry's material loop [47,50]. Collection and sortation are activities that divert post-consumer waste from landfills. For garments sent for reuse, it allows to select items that meet sanitation and quality requirements for thrift shops. Whereas, they are crucial for enabling the viability of recycling operations, as it preconditions waste for specific processes.

Another barrier is that garments are made with different blends of fiber materials and other components, such as buttons and zippers, which hinders their recyclability potential [3]. Hence, the valorization of textiles through recycling processes is still limited by technological and operational reasons [41, 47]. Additionally, consumers have a lower emotional value attached to their clothing which affects their perception of the garment when they want to dispose of it. It compromises the reuse or recycling of an end-of-life product because the consumer will not consider that it could still hold some value [51].

Secondhand-market's reuse and recycling processes, dependent on brands and retailers' ability to retrieve worn garments through reverse logistics, are examples of end-of-life strategies needed in driving and maintaining a sustainable transition in the apparel industry. For such results, stakeholders of the supply chain need to be supported by policymakers committed to put incentives in place and reduce barriers.

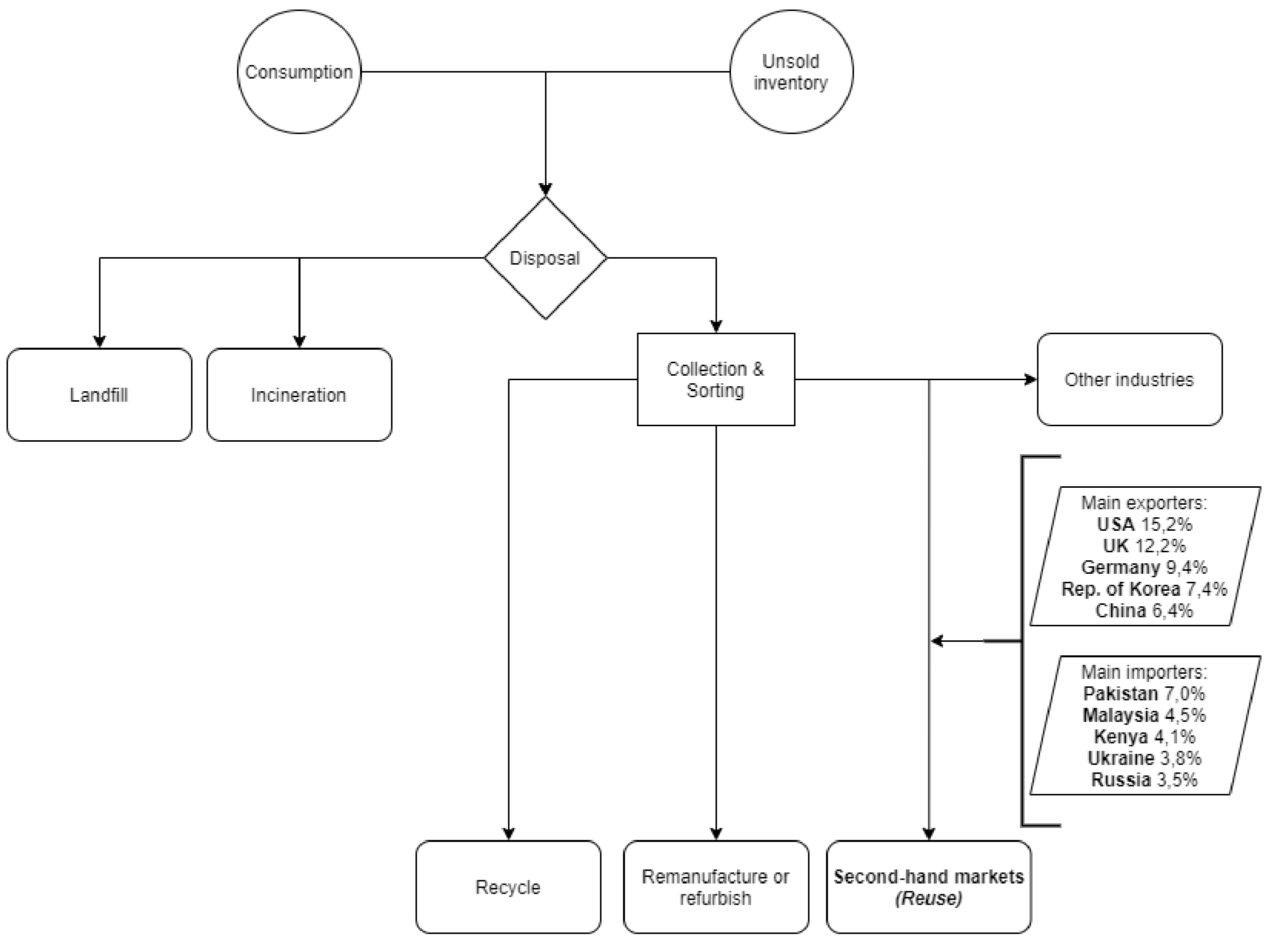

Figure 2.4 Worn garments flow.

Note: Trade flow data from International Trade Center, Trade Map - Canada, retrieved online in August 2020

2.6 Public Policies

Jia et al. (2020) produced a systematic literature review highlighting barriers and drivers for circularity in the apparel industry. One of the main findings is how governments can exercise an increased incentive to lead businesses into a sustainable transition. Especially in the absence of a structure of channels for circular strategies in textiles, policymakers can adopt regulations and environmental policies that reduce costs for businesses willing to dispose appropriately of their textile waste [26]. The different governmental strategies to address environmental issues linked to the apparel industry take many forms, from taxation to an extended producer responsibility program (EPR) to finance waste management. Kelderman (2019) highlights the importance of policymakers by listing and analyzing such public policies that could support an increase in recycling textiles.

On an international level, there is also an important recognition of the industry's problems. Indeed, one of the two UNFCCC's sectorial initiatives is dedicated to the apparel industry under the Fashion Industry Charter for Climate Action [52]. It invites industry leaders to coordinate in developing a roadmap for a sustainable transition that follows the Paris Agreement's decarbonization. The publications created by this Sectoral Engagement and the conversations it has started aim directly at firm's strategies and what companies can do to improve their supply chain's environmental performance. Yet, it remains a voluntary commitment from firms.

In terms of examples of environmental policies, the European Union already adopted policies to promote circularity in waste management for member countries [26]. Notably, before this new EU policy, France spent years building the framework for an EPR-type legislature [53] to deter retailers from disposing of unsold garments in landfills or incineration centers. It is key to support closed-loop alternatives.

An EPR policy shifts part of the costs of end-of-life treatment towards producers and retailers to finance disposal alternatives available in a territory. In addition, Refashion, the French organization responsible for the valorization of worn garments in the country, makes available a knowledge base and cheat sheets for ecodesign strategies. Ecodesign, which occurs on the drawing table, helps promote preferred materials, certifications that influences downstream manufacturers, and reduce the product's overall environmental impact. It also plays a significant role in facilitating the preconditioning of garments for recycling by encouraging detachable components or a homogeneous material choice. This way, they encourage and support brands in the development of products and services that lessen the overall environmental impact of a garment by applying sustainable design choices that consider the product lifecycle.

Similarly, Sweden and the United Kingdom have different types of policies and programs either already in place or planned for the near future. In particular, Sweden recently proposed a tax on apparel containing hazardous chemicals, either manufactured in Sweden or imported, with the aim of "cost-effectively [reducing] the incidence or risk of exposure to, and spread of, substances in clothing and footwear that are harmful to the environment and human health" [54]. Similarly, the UK will impose a tax on plastics in 2022, and they were recommended by experts to expand the criteria to include polyester and other petrochemical components used in garments [3].

Canada signed the Paris Agreement and the Agenda 2030 that establishes the Sustainable Development Goals, one of which is responsible consumption and production [55]. That commitment and others international and national stances justify investing resources to deter retailers and brands from destroying product returns and unsold inventory. If nothing is done in the apparel industry, the targets that Canada committed to achieve in order to mitigate climate change cannot be expected to be met.

In 2009, the Canadian Council of Ministers of the Environment produced the Canada-wide Action Plan for Extended Producer Responsibility to advise and coordinate EPR policies for provinces on priority products [56]. It comprises two phases and, today, Phase 1 products such as packaging, electronics and automotive components have provincial recycling programs [56, 57] . However, textiles were part of an eventual Phase 2 for 2017 as the sector was deemed not ready yet. The recommendation was a timeline of 8 years to fill the lack of data, technology, and linkage between stakeholders [58]. In 2021, only British-Columbia includes textiles in a future EPR plan [59] and Ontario has the Ontario Textile Diversion Collaborative, which is a non-profit organization promoting municipal initiatives with the notable example of the City of Markham’s Textile Recycling Program Strategy [60]. The initiative puts in place collection and sorting operations to avoid that post-consumer clothing is sent to landfills.

The disposal alternatives for garments, in Canada, remains in majority the reuse through companies and nonprofit organizations. Reuse might seem like a good strategy since it is higher in the waste management hierarchy. However, 75% to 80% of donations do not meet the requirements for reuse by secondhand markets [61]. Even if exported, these garments can remain unfit for reuse and end up in landfills around the world. Hence, there is a need for scaled-up viable recycling options to treat them and to deter retailers from destroying their end of season garments. Fashion Takes Action, a Canadian organization that promotes awareness and solutions for sustainability issues in the fashion industry, recently published “A

Feasibility Study of Textile Recycling in Canada" that goes in a similar direction of analysis, conclusions and recommendations [62].

Considering retailers and brands burn their unsold inventory and returned products, further analysis of available tariffs and regulations is required to avoid creating those additional incentives that would promote the destruction of products. Especially in the case of incineration for energy recovering, a company could even receive a tax benefit for destroying inventory [28].

2.7 Conclusion

Apparel consumption is responsible for environmental damages and social issues that have rightfully been on the spotlight of discussions on the sustainability of consumable goods. Fast fashion is perceived to heavily contribute to these problems with its high speed of new trend cycles and low-quality garments. The impact is two-fold with an aspect of overconsumption and, yet also an overproduction. In the management of the excess stock at the end of a selling season, retailers and brands have incentives to destroy it instead of recovering its value. The elimination of unsold inventory is a strategy closely linked to the linearity of practices related to fast fashion.

In trying to understand why some firms decide to destroy their unsold inventory, the advantages of fast fashion for retailers and brands were highlighted. Namely, a faster supply chain enabling the design of a garment which fashion content's is closer to the present trend. Indeed, one of the goals of a fast fashion brand or retailer is to quickly bring to market a product that will satisfy the maximum number of consumers for a selling season. One downfall is the production of lower quality garments that have low value when the trend fades. The nuances of this rapid product obsolescence and the demand uncertainty add to the issues of the linear model of the current apparel industry. It contributes to the generation of excess stock that retailers and brands need to dispose of in order to free in-store space for new apparel collections. The decisions taken during production according to fast fashion requirements aggravate the post-consumer waste management issues since available end-of-life treatments are similar for unsold inventory and worn garments.

Textile waste issues have become global, and they contribute to the already heavy environmental and social impacts of the apparel industry. Unsold inventory directed to landfills and incineration centers worsen waste management challenges. Hence, there is a need for a systemic adoption of solutions both for better production practices and end-of-life alternatives. To further understand where these strategies can be applied, an overview of the fashion industry supply chain was conducted. It highlighted how opaque access to data is and how retailers have become the new key decision-makers.

The globalized and fragmented characteristics of the apparel industry become evident when tracking flows of main materials and fabrics used to manufacture garments. It shows how North American, and some European countries have become "consumers" as finished products are imported in greater quantities than anywhere else. However, the Harmonized System codes used for analyzing international trade are limited when trying to understand textile waste flows, which hinders the comprehension of end-of-life impact. Generally, unsold inventory does not cross borders, because it is treated by local or regional disposal facilities, commonly landfills and incineration centers rather than recycling operations. There is the exception of reuse when unsold products are sent to NGOs and second-hand stores as corporate donations. Although, if they do not meet the criteria to be sold in other national second-hand markets, these end up as waste around the world. Globally, there is a general lack of data on textile waste. However, available data shows that textile waste is a problem requiring actions. Hence, the first steps for efficient policymaking would be to facilitate tracking of waste when it crosses borders through new types of commodity codes and to further undergo waste characterization studies. The latter is a localized and small-scale effort, but both are needed to have a clearer portrait of the situation to develop appropriate recovery solutions.

As retailers became the central stakeholders thanks to their closeness to consumers' feedback and private labels, their responsibilities have been increased regarding production and disposal of garments. There is a need to reduce production quantities of low-quality garments that are incompatible with existing recycling processes. Also, ecodesign strategies, choice of materials and low-impact processes are required to reduce the overall impact of the industry

Opportunities for closed-loop strategies exist but have yet to be applied into large scale operations. Significant technical barriers strengthen the industry's linearity by preventing these initiative's growth. Fabrics made of blended fibers, the most widespread collection method, and manual sorting of garments result in batches of heterogenous materials, which jeopardize recycling operations. These obstacles generate a larger volume of residual waste because landfills are more available, and often less costly, than recovery alternatives. New business models are needed in the fashion industry, with a focus on cleaner production and consumption that reduces waste. This transition needs governmental support to allow better practices to become the new industry standard.

CHAPTER 3 GENERAL APPROACH

3.1 Return on Chapter 2

The article in Chapter 4 builds on the main findings from the working paper presented in Chapter 2, which are the fashion industry environmental impact, the link between fast fashion and the disposal of unsold inventory, and existing public policies.

Mainly, in the industry, production operations are the major source of environmental impact on climate change, ecosystems quality and land use. This impact is exacerbated by fast fashion due to faster trend cycles and a push for newness. For the specific case of unsold inventory, these are never worn clothing that generate an impact during production and, now, require a disposal. Mostly, retailers direct this excess stock towards the same disposal methods available for post-consumer clothing. However, landfills and incineration are the main treatment as more sustainable methods are either saturated or in small scale. Hence, there is a need to propose solutions that act on the production side and to implement legislation in consumer countries to influence production.

Through economic incentives, public policies are used to internalise the costs of negative externalities causing social and environmental damage. This has a direct effect on retailers' decision-making regarding quantities brought to market. The Chapter 2 highlights some types of regulations currently, or in the near future, in place around the world. From these, the article on Chapter 4 selects two public policies to explore their effect on quantities brought to market. The first is a tax on disposal of unsold inventory, which aims to deter retailers from destroying their surplus. The second is an EPR fee applied to the total quantity brought to market, with the goal of increasing total production costs.

In Chapter 2, it is mentioned how the literature addresses the fast fashion inventory management issues through the lens of the newsvendor problem and that excess inventory at the end of the season occurs due to miscalculations (sections 2.2 and 2.3). The literature on the newsvendor problem highlights multiple scenarios where the decision-maker has risk-preferences that motivates them to increase their safety inventory in order to avoid missing sales opportunities. Although a popular argument, it contradicts others' argument of miscalculations as it gives intent to larger inventories. The newsvendor problem literature is relevant considering the features of uncertainty in the fast fashion context, but the model developped in Chapiter 4 is distinguished due to our attempt to focus on the newness — or the consumers willingness to pay for newness [63] — and substitutability between the novelty and end of the season clothing. Indeed, the newsvendor problem is a way of addressing fast fashion inventory management issues, but, instead, the model developed in the article focuses on the newness and substitutability aspects.

3.2 Article Method

The literature that addresses the inventory management of fast fashion builds on the newsvendor problem, with risk-preference variations and a stochastic demand. We depart from this approach to consider economic incentive policies and, notably, consumers willingness to pay for the novelty prompted by fast fashion trends. This latter consideration is an original contribution of the article to the literature on fast fashion, as it further explores the influence of trends on both consumers and retailers. Overall, the article explores the effect of consumers willingness to pay, the cross-elasticity between fashionable products and additional costs introduced by a tax on disposal of unsold inventories and a fee from an EPR-type legislature for products brought to market.

Through the development of an original optimization model for sub-objective 2, the construction decisions include the effects of fast fashion. Noticeably, the model considers a retailer's up front decisions about their market positioning, how closely they follow fashion trends and consumer's willingness to pay for novelty. Also, the construction of the model's inverse demand allow to explore the substitutability between fashionable clothing at the start of the season, sold at regular price, and the same product on sale at the end of the season.

Then, the model establishes the optimization of two different equations. Initially, it is a monopolistic retailer's production problem, where they first decide on a quantity related to fashionable trends before deciding on the rest of their production. Afterwards, this retailer's production equilibrium is compared with the social optimum of a social planner maximizing a social welfare function. In particular, that social welfare function departs from convention by including the environmental damage caused by externalities in addition to the standard consumer surplus, profits and tax revenues.

Sub-objective 3 uses the equilibrium found in the retailer's production problem and the social optimum to estimate the effect of parameters and public policies. The studied parameters are consumer's willingness to pay for novelty, and the cross-elasticity between products. While the public policies of interest are a tax on disposal and an EPR fee.

A comparative static analysis is used to observe how the parameters and public policies affect the social welfare and the decision-making of a retailer. This type of analysis require to take the partial derivative of elements of a function in respect to the targeted parameter. The result gives an insight on the direction of the change to an element when there is an increase of the parameter or economic incentive.

Hence, the article on Chapter 4 fits in with the goal of this research project by exploring how consumers desire for novelty and public policies can influence the fashion industry externalities while considering the main effects of fast fashion.

CHAPTER 4 ARTICLE 1: FAST FASHION: WHY FIRMS INCINERATE DEADSTOCK, AND PUBLIC POLICIES

Pedro Cybis, Sophie Bernard. (2020). Fast Fashion: Why Firms Incinerate Deadstock, and Public Policies. Article accepted on May 13, 2022 at the 2022 annual congress of the Société canadienne de science économique.

4.1 Abstract

Fast fashion increases the overall consumption and need for new products each season. With trends rapidly fading away, remaining garments lose their value and occupy precious space in stores. The destruction of this unsold inventory contributes to the already significant environmental impact of the fashion industry. This article develops an optimization model that proposes a demand function including consumers desire for newness and the cross-elasticity of high fashion content clothing at the start of the season and the same product on sale at the end of the trend. At first a retailer solves a production problem with the presence of public policies: a tax on disposal of unsold inventory and an extended producer responsibility fee. Then, the quantities brought to market in this equilibrium are compared to a social optimum to highlight the effect of economic incentives on inventory. As additional costs to the retailer, there is an expected decline of the total quantity produced. Afterwards, a comparative static analysis is carried out for the parameters. Notably, consumers willingness to pay for novelty puts pressure on retailers to bring to market fashionable collections each season, which pushes a price increase to support their production. A lifecycle mindset establishes the relevance of addressing issues along the supply chain. Public policies are critical to support solutions from processes to garment value recovery. As unsold inventory is expected by firms, there is a need for policymakers to reduce incentives and existing mechanisms leading to deadstock destruction.

4.2 Introduction

Apparel, being the third largest manufacturing industry worldwide [1], has a significant environmental impact. This is attributed to the production phases and the linearity of apparel's lifecycle [19,21]. Indeed, the alternatives to final end-of-life disposal treatments are limited; circular solutions are still operated on a small scale or in early development [16]. Landfills and incineration centers are among the conventional disposal methods available for post-consumer and pre-consumer waste, which includes unsold inventory.

The volume of unsold inventory increases with time as the fast fashion phenomenon spreads. Fast fashion exaggerates the speed of trend cycles, making clothing with higher levels of fashion content obsolete at the end of the season [13]. Due to the quicker lead time to market imposed by the supply chain structure of fast fashion, apparel in this market segment are lower quality products. Another effect of fast fashion is an increase of the desire for novelty. Together, these effects increase the quantity of unsold apparel due to larger initial inventories, which exacerbates the industry's environmental impact of greenhouse gas (GHG) emissions, water footprint and waste generation of the industry [3, 19, 21].

Whereas in a more conventional market the brands' decision-makers were the most important of the industry, fast fashion shifted this power towards large retailers [14]. Retailers' private label collections better match trends due to their more integrated supply chains and the closeness with customers' feedback [14]. These advantages and autonomy have put retailers in an ideal position to influence the industry. Indeed, these are the stakeholders with the most influence on the production of clothes behind the overconsumption of fast fashion. Retailers order the quantity produced and choose to have an excess inventory at the end of the season. In this position, they are also the ones who have the power to change end-of-life disposal methods.

It is based on this assumption of deliberate choice by retailers that this article examines how economic incentives provided by public policies can limit the environmental impact of the fashion industry. Apparel, particularly in fast fashion, displays characteristics similar to the classic newsvendor problem in economics (i.e. newness, short lifecycle and demand uncertainty). In its classic form, the newsvendor problem solution is an ordered quantity of a new product according to a decision-maker with a neutral risk-preference. The quantity produced must be decided before the start of the period and there are no inventory modifications possible once customers start purchasing. The inventory decisions in the fashion industry follow a similar constraint due to its globalized production.

This limit on inventory management highlights the need for a buffer inventory [11]. Relatively recent literature shows that distinct risk-preference profiles result in different-sized inventories [13,36,39,40,64,65]. In particular, there are scenarios where a risk-averse decision-maker, such as fast fashion retailers, will prefer a surplus inventory to increase the buffer inventory [40,65]. It is done to avoid a stockout situation where there are losses of sales opportunities, customers loyalty and market competition. Hence, the retailer decides to order a larger inventory than the expected demand. This difference leads to a deliberate surplus that is the unsold inventory remaining at the end of the season.

This article departs from the newsvendor problem, while acknowledging its relevance and insights, and introduces stylized assumptions reproducing consumer behavior in order to take into account the role of the demand for novelty and clothing's substitutability between the start of the trend and its end.

This article proposes an original model representing the effect of consumers' willingness to pay for novelty on the optimization of ordered quantities for a retailer. The goal of the model is to take in consideration this consumer need for newness and its effect on ordered quantities by a retailer. The three types of agents present are the consumers, a retailer and a social planner such as a government. The model explores three approaches to examine how public policies can reduce the environmental impact of unsold inventories. Initially, the producer problem is developed where a retailer decides on quantities brought to market, but needs to take into account additional costs for the disposal of unsold inventory. Then, a welfare analysis takes a look first on the quantities of a social optimum and, lastly, a comparative static analysis of key parameters and the variables related to the policies. The chosen public policies are economic incentives such as a tax for the disposal of unsold inventory and an extended producer responsibility (EPR) fee for products brought to market.

The structure of the article continues with a short literature review to further situate fast fashion, the lack of alternatives for the disposal of unsold inventory and relevant public policies. Then, the article proceeds with the original optimization model. Finally, a comparative static analysis is done, and a discussion highlights the main findings.

4.3 Literature Review

There are other industries than fast fashion that face challenges related to the novelty aspect of their product. Notably, textbook publishers are known to release new editions in order to kill off older versions [66]. Iizuka (2007) demonstrates how the presence of an used textbook market motivates publishers to find the optimal moment for an update based on the stock of used textbooks and the number of years between editions. Here, the newer edition devalues the older ones, much like new trends in fast fashion. It is a case both of planned obsolescence, where the value of the initial product is diminished after the sale, and of crippling secondary markets.

Also of relevance, the music industry is constantly introducing singles and albums that have a novelty or innovative aspect and that loose a part of their value with time as less consumers show willingness to pay [67]. Interestingly, to ensure a consumer pool, the industry relies heavily on advertisements to create attention from "advertising externalities" that lets consumers know about a single preparing for an album [67]. In particular, informative or persuasive advertisements are expenses that increase firms profits [68–70].

4.3.1 Fast Fashion

Apparel has two main aspects: function and aesthetics [14]. For the aesthetic aspect, apparel is subject to varying levels of fashion content (i.e. what is considered trendy in the style) ranging from high fashion to basic products [12, 14]. As an example, basic levels are clothing such as simple black t-shirts or blue jeans, while a high fashion content level will have characteristics specific to the current trend. Therefore, high fashion garments loose their value at the end of the season once the trend fades.

These function and aesthetics aspects create a seasonality constraint on apparel [14]. An example of seasonal functions are summer versus winter clothing, while aesthetics follow the trends. The general implications of this seasonality are that apparel has short life-cycles subject to high demand uncertainty due to changing trends, to forecasting obstacles and to consumer's impulsive purchase behavior [27]. These issues are similar to those of other consumption goods, but drive purchasing behaviors to overconsumption in the context of fashion.

First, the seasonality of apparel contributes to the demand uncertainty and forecasting obstacles [27]. In the fast fashion context, the recent literature blames the surplus of unsold inventory on these uncertainties that cause miscalculations when ordering inventory [28]. To counter those risks, fast fashion retailers are known for making their supply chain faster and more flexible, even becoming case studies for quick response, just-in-time and agile supply chains [13, 14].

Second, apparel has a short life-cycle [27], especially, with the fading of trends being a major contributor in the context of fast fashion. Also, due to the pressure for fast production and delivery that compromises quality control processes, fast fashion apparel has lower quality [13, 51]. The short life-cycle of fast fashion apparel contributes to its overconsumption by limiting the use phase of clothing and its possible reuse.

Lastly, as a consumption good, apparel induces impulsive purchase behaviors from consumers, which contributes to the difficulties of ordering accurate quantities [27]. An effect of fast fashion on consumers is the scale up of the desire for novelty. Indeed, the changing fashion trends form through cultural shifts that influences popularity and consumer's demand that heighten their desire for newness [15]. Through fast fashion, impulsive purchase increases and is a part of retailers strategy to reach their customers.

Brands were the main decision-makers in the fashion industry [14]. With fast fashion, retailers have developed strategies such as their own private brands and direct relationships with manufacturers in the supply chain, resulting in faster lead times to market [13, 14, 71].

4.3.2 Pre-Consumer Waste

The surplus left after the trend fades needs to be managed because the apparel looses its value and the retailer needs to make space in stores for the next new collection. These never-worn products become pre-consumer waste, and are directed to the same disposal channels as post-consumer waste. Worldwide, the vast majority is sent to landfills and incineration centers that are virtually cost-less [3, 16]. Still, reuse through the secondhand market, like for post-consumer clothing, is available for pre-consumer waste in the form of corporate donations. However, this is a limited disposal alternative for two reasons.

First, in general, brands and retailers prefer to destroy their unsold inventory instead of giving them away to protect their brand image and ensure a scarcity [28]. Having products sold at a very low price compared to the regular price can be detrimental to a brand. However, in the case of fast fashion retailers, there is less an image of elitism to portray and more a convenience of available disposal methods.

Secondly, the global reuse market is saturated, causing the value of pre-consumer and post-consumer clothing to remain low [3]. This is a major barrier to the financial viability and scale up of collection and sorting operations.

Hence, unsold inventory is treated as pre-consumer waste, which is sent to landfills or incinerated. Reuse is the best alternative readily available, but its operational capacity is limited. While recycling clothing into new textiles is still in early stages of development. A focus on disposal process is needed to understand the volume of waste from the fashion industry, but also to scale up the alternative solutions.

4.3.3 Public Policies

Public policies can support the development of collection, sorting and recycling through investments to grow and promote those operations [26]. However, a focus on disposal processes lacks an effect on production volumes of the fashion industry. Indeed, there is a need to address the scale of new apparel being brought to market that will, for the majority, end up in landfills. Economic incentive policies used as targeted additional costs can influence the initial quantities in stores in order to reduce the surplus leftover.

There are many types of existing public policies and environmental regulations that can be adapted to the fashion industry. Kelderman [23] brings together multiple policies that have a potential to inhibit the industry's environmental impact and highlights the most feasible and impactful ones. The recommendation that Kelderman identifies with the highest priority is a mandatory EPR.

An EPR would make retailers responsible for the disposal of products they bring to market. While it adds an extra cost, funds accumulated are invested on the treatment infrastructure for pre-consumer and post-consumer waste. It is a way of supporting collection and sorting operations, while promoting alternatives to landfills and incineration. An example is France that was the first country to apply an EPR-type legislation to the fashion industry. It restricts retailers and brands on how they can dispose of their unsold inventory. They have an obligation to ensure that it is either sent to secondhand markets or to recycling operations. However, the reuse option has a risk of facing the saturated global market and being disposed of in landfills around the world.

Another example of regulations are economic policies such as a bonus-malus system or the refunded virgin material payments introduce an additional cost to products [23]. Notably, the destruction of unsold inventory is less expensive than an alternative disposal solution [26]. Hence, a tax on disposal can add expenses that deter the use of landfills for deadstock.

4.4 Model

A monopolist, retailer of a leading brand, produces fashion apparels. The retailer's decisions consist of three quantities: a quantity related to trends w, a quantity sold at regular price q_r and a quantity sold at a discount price q_s.

Much like in Iizuka (2007) textbook publisher model, the choice of modeling a monopolist is based on the control of the firm on the useful life of the product. Indeed, each trend is introduced by a specific brand that has a complete control over it and the length of time for which the trend is relevant. This way, the retailer builds their brand image and has a monopolist power over it compared to a less known brand.

The up front decisions regarding trends and the level of fashion in the collections are translated into the quantity related to trends w. A larger w is the result of more numerous and trendy collections, while a retailer with basic apparel collections will display a smaller w.

The quantity related to trends w is distinct from the sold quantities q_r and q_s, because w is meant to lure more customers into stores instead of just being sold; it acts like advertisement spending on market size. The full quantity produced will be $(q_r + w)$, meaning that when $(q_r + w)$ is produced, only q_r is expected to be sold at regular price. The quantity sold at discount price should respect the constraint $q_s \leq w$. Hence, the total quantity sold is $(q_r + q_s)$, and the difference $(w - q_s)$ at the end of the selling season represents the unsold inventory. The retailer is forced to dispose of this inventory, because they need store space for the next trend.

Production costs take the following form: $c(q_r + w^2)$, with c a constant. We assume that basic collections are made at constant unit cost, whereas trendier collections require quick response and more intensive supply chains, leading to increasing marginal costs.

The retailer may be subjected to two environmental policies: a disposal tax τ_1 on unsold inventory $(w - q_s)$, and an extended producer responsibility program applying an ecotax τ_2 on all products brought to market $(q_r + w)$.

A regular price, p_r, is set at the start of a selling season, while at the end of the season items are sold at p_s. Someone who values being trendy displays a willingness to pay a regular price for apparel that is new and with a higher level of fashion content w. However, consumers will also predict the reduction of prices during sales and be willing to pay less even if the trend would fade sooner. New apparels and apparels at the end of the season will be considered as imperfect substitutes. Then, the inverse demand functions are expressed as:

$$p_r = \alpha_1 + \alpha_2 w - \beta_1 q_r - \gamma q_s \tag{4.1}$$

$$p_s = 1 - \gamma q_r - \beta_2 q_s \tag{4.2}$$

Let PD_P be the difference $PD_P = \beta_1\beta_2 - \gamma^2$, an index of product differentiation. Products are independent when $PD_P = \beta_1\beta_2$ and perfect substitutes when $PD_P = 0$. In the present paper, we will investigate cases of imperfect substitutes, when $0 < PD_P < \beta_1\beta_2$.

The parameter α_1 stands for the total pool of consumers with a need to buy clothing at the beginning of the season, and α_2 is the consumers willingness to pay for the novelty of a trend related to w.

This one period model is static because fast fashion is based on short life-cycles due to the fashion trend being developed specifically for that time period. Due to a planned obsolescence, there is no secondary market for apparel for which the trend it is a part of has faded. If it wasn't the case, then a dynamic model would be preferable as used clothing would compete with new ones. As the model aims to explore the effect of consumers willingness to pay for newness, the only relevant period is the one for which the trend is valid. Here, consumers purchase out of a need for novelty instead of a basic need for clothing.

The Retailer's Production Problem

The retailer maximizes profits π by optimizing the quantity related to trends w, quantity sold at regular price q_r, and quantity sold on sale q_s. The profit function is:

$$\pi_{(q_r,q_s,w)} = p_r q_r + p_s q_s - c(q_r + w^2) - \tau_1(w - q_s) - \tau_2(q_r + w) \tag{4.3}$$

In a first stage, the retailer will commit to a trend w. The choice of w is based on the retailer's market positioning that reflects the brand image, and can be interpreted as a long term commitment. In a second stage, q_r and q_s are chosen. Environmental policies τ_1 and τ_2, as well as inverse demands p_r and p_s (eq. 4.1 and 4.2) are taken as given. The problem is solved backward. In the second stage, we have:

$$max_{q_r,q_s}\ \pi$$
$$\text{subject to } q_s \leq w,$$

which can be written as:

$$\mathcal{L}_{(q_r,q_s,\lambda)} = \pi_{(q_r,q_s)} + \lambda(w - q_s)$$
$$\mathcal{L}_{(q_r,q_s,\lambda)} = p_r q_r + p_s q_s - c(q_r + w^2) - \tau_1(w - q_s) - \tau_2(q_r + w) + \lambda(w - q_s)$$

The Karush-Kuhn-Tucker (KKT) conditions are:

$$\frac{d\mathcal{L}_{(q_r,q_s,\lambda)}}{dq_r} = \alpha_1 + \alpha_2 w - 2\beta_1 q_r - 2\gamma q_s - c - \tau_2 = 0$$

$$\frac{d\mathcal{L}_{(q_r,q_s,\lambda)}}{dq_s} = 1 - 2\gamma q_r - 2\beta_2 q_s + \tau_1 - \lambda = 0$$

$$\frac{d\mathcal{L}_{(q_r,q_s,\lambda)}}{d\lambda} = \lambda(w - q_s) = 0, \lambda \geq 0$$

Optimal values are:

$$\bar{q_r}(w) = \frac{\beta_2(\alpha_1 + \alpha_2\bar{w} - c - \tau_2) - \gamma(1 + \tau_1 - \bar{\lambda})}{2(\beta_1\beta_2 - \gamma^2)} \tag{4.4}$$

$$\bar{q_s}(w) = \frac{\beta_1(1 + \tau_1 - \bar{\lambda}) - \gamma(\alpha_1 + \alpha_2\bar{w} - c - \tau_2)}{2(\beta_1\beta_2 - \gamma^2)} \tag{4.5}$$

Whereas optimal prices (regular and on sale) $\bar{p_r}(w)$ and $\bar{p_s}(w)$ become:

$$\bar{p_r}(w) = \frac{\alpha_1 + \alpha_2 \bar{w} + c + \tau_2}{2} \tag{4.6}$$

$$\bar{p_s}(w) = \frac{1 - \tau_1 + \bar{\lambda}(w)}{2} \tag{4.7}$$

It is interesting to note that $\bar{p_s}$ will depend on w through the Lagrangean multiplier $\lambda(w)$.

In the first stage, the retailer will choose the quantity related to trend w while taking (4.4) to (4.7) as given. We have:

$$\begin{aligned} max_w \; \pi(w) =& \bar{p_r}(w)\bar{q_r}(w) + \bar{p_s}\bar{q_s}(w) - c(\bar{q_r}(w) + w^2) \\ & - \tau_1(w - \bar{q_s}(w)) - \tau_2(\bar{q_r}(w) + w) \end{aligned}$$

The first order condition is:

$$\frac{d\pi_{(w)}}{dw} = \frac{\partial \pi}{\partial p_r}\frac{\partial \bar{p_r}(w)}{\partial w} + \frac{\partial \pi}{\partial p_s}\frac{\partial \bar{p_s}}{\partial w} + \frac{\partial \pi}{\partial w} = 0$$

Because we are interested in unsold inventories, i.e.: $w - q_s > 0$, we will treat cases of a slack constraint, with $\lambda = 0$. This ensures that the model represents the main topic intended by this study. We hence have:

$$\frac{d\pi_{(w)}}{dw} = \frac{\alpha_2[\beta_2(\alpha_1 + \alpha_2 w - c - \tau_2) - \gamma(1 + \tau_1)]}{4(\beta_1\beta_2 - \gamma^2)} - 2cw - \tau_1 - \tau_2 = 0$$

$$\bar{w} = \frac{\alpha_2[\beta_2(\alpha_1 - c - \tau_2) - \gamma(1 + \tau_1)] - 4(\beta_1\beta_2 - \gamma^2)(\tau_1 + \tau_2)}{8c(\beta_1\beta_2 - \gamma^2) - \beta_2\alpha_2^2} \tag{4.8}$$

Additionally, this result respects the second order conditions[1].

For simplicity, in addition to PD_P as the index of product differentiation for the retailer's problem and RM_P is the regular margin:

$$RM_P = \alpha_1 - c - \tau_2$$

The regular margin RM_P can be interpreted as the potential margin from goods at regular

[1] The second derivative shows the condition for which $\bar{w}$ is a maximum under the constraint $\frac{\beta_2\alpha_2^2}{4(\beta_1\beta_2 - \gamma^2)} < 2c$.

price.

By substituting (4.8) into optimal quantities and prices, equations (4.4) to (4.7), we obtain:

$$\bar{w} = \frac{\alpha_2[\beta_2 RM_P - \gamma(1+\tau_1)] - 4PD_P(\tau_1+\tau_2)}{8cPD_P - \alpha_2^2\beta_2} \tag{4.9}$$

$$\bar{q_r} = \frac{2[2c[\beta_2 RM_P - \gamma(1+\tau_1)] - \alpha_2\beta_2(\tau_1+\tau_2)]}{8cPD_P - \alpha_2^2\beta_2} \tag{4.10}$$

$$\bar{q_s} = \frac{\beta_1(1+\tau_1) - \gamma RM_P}{2PD_P} - \frac{\alpha_2\gamma}{8cPD_P - \alpha_2^2\beta_2}\left[\frac{\alpha_2[\beta_2 RM_P - \gamma(1+\tau_1)]}{2PD_P} - 2(\tau_1+\tau_2)\right] \tag{4.11}$$

$$\bar{p_r} = \frac{(RM_P)(16cPD_P - \alpha_2^2\beta_2) - 2\alpha_2[\alpha_2\gamma(1+\tau_1) + 4PD_P(\tau_1+\tau_2)]}{4(8cPD_P - \alpha_2^2\beta_2)} \tag{4.12}$$

$$\bar{p_s} = \frac{1-\tau_1}{2} \tag{4.13}$$

Proposition 1 *The expressions of quantities and prices at the private optimum are the equations from 4.9 to 4.13.*

4.4.1 Social Optimum

In the producer problem, the retailer optimizes his profit function by choosing the quantity variables. However, this function does not take into account environmental damage and the social welfare of consumers. To include them, it is interesting to introduce a social planner, such as a government, aiming to optimize a welfare function rather than a profit function.

Social Welfare Function

Consumer Surplus As with any other consumption good, the quantity of apparel brought to market by the retailer contributes to the social welfare. For customers, their welfare from purchasing new clothing is represented by the consumer surplus (CS).

$$CS_{q_r} = \int_0^{q_r} [p_r(q_r) - \tilde{p}_r]\, dq_r$$

$$CS_{q_s} = \int_0^{q_s} [p_s(q_s) - \tilde{p}_s]\, dq_s$$

$$CS = \int_0^{q_r} [p_r(q_r) - \tilde{p}_r]\, dq_r + \int_0^{q_s} [p_s(q_s) - \tilde{p}_s]\, dq_s \tag{4.14}$$

with $\tilde{p}_r$ and $\tilde{p}_s$ given values of regular price and price on sale.

Externalities Conventionally, the social welfare (SW) comprises the consumer surplus and the producer surplus. The latter are captured by the profit function that comprises the revenues and production costs. Also, the tax τ_1 and EPR fee τ_2 that are perceived from the retailer's profits return to increase the social welfare, nullifying the effect of those additional costs. However, an important modification is made to the conventional social welfare function in order to consider the environmental damage from externalities occuring in production.

Normally, the retailer's optimization process of their profit function does not internalize the damages of negative externalities caused by the environmental impact during production. To address this gap, the model includes an externality element, D, in the social welfare function SW where D is a constant. More than harming ecosystems, environmental damage D has also an impact on communities near manufacturers as it affects human health. These effects can be assigned a monetary value useful to consider in the social welfare.

$$SW = CS + Profits + TaxRevenues - Externalities$$

$$SW = CS_{q_r} + CS_{q_s} + \pi + \tau_1(w - q_s) + \tau_2(q_r + w) - D(q_r + w)$$

$$\begin{aligned} SW = &\int_0^{q_r} [p_r(q_r) - \tilde{p}_r]\, dq_r + \int_0^{q_s} [p_s(q_s) - \tilde{p}_s]\, dq_s \\ &+ p_r q_r + p_s q_s - c(q_r + w^2) - D(q_r + w) \end{aligned} \tag{4.15}$$

Results

The social planner decides the value for the quantity variables w, q_r and q_s in order to achieve the social optimum. This equilibrium is also known as *first best*, as it maximizes the social welfare SW. This equilibrium between the quantities is interesting to compare with the quantities expressions found on Proposition 1 of the producer problem.

If it is impossible to the social planner to set up quantity variables similar to the *first best*, they can introduce regulations to influence the retailer's decision-making. The value of the economic incentives from public policies is the *second best* equilibrium. In this model, this equilibrium comes from the social planner's maximization of the social welfare function SW by deciding on the values of the tax τ_1 and the EPR fee τ_2. However, the second best optimum will not be explored here.

First Best The optimization of the social welfare equation SW by the social planner proceeds through the same steps as the optimization of the profit function by the retailer. Also, the constraint $q_s \leq w$ remains in order to ensure that the consumption is upper-limited to the total production $(q_r + w)$. However, unlike the retailer's problem, all three quantity variables (w, q_r and q_s) are decided simultaneously.

$$max_{w,q_r,q_s}\, SW$$
$$\text{subject to } q_s \leq w.$$

Then, there is a need for a Lagrangean multiplier due to the constraint on quantities sold on sale q_s and the Lagrangian is:

$$\mathcal{L}_{(w,q_r,q_s,\lambda)} = CS_{q_r} + CS_{q_s} + \pi + \tau_1(w - q_s) + \tau_2(q_r + w) - D(q_r + w) - \lambda(q_s - w)$$

Which gives the following KKT conditions:

$$\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{dw} = \alpha_2 q_r + \lambda + 2cw - D = 0$$

$$\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{dq_r} = \alpha_1 + \alpha_2 w - \beta_1 q_r - 2\gamma q_s - c - D = 0$$

$$\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{dq_s} = 1 - 2\gamma q_r - \beta_2 q_s - \lambda = 0$$

$$\lambda\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{d\lambda} = \lambda(w - q_s) = 0, \lambda \geq 0$$

Similarly to the retailer's production problem, KKT conditions are introduced to take into account the inequality constraint $q_s \leq w$. The scenario where unsold inventory is present happens when the KKT conditions are inactive (λ=0 while $q_s < w$). Otherwise, when the KKT conditions are binding the Lagrangean multiplier $\lambda > 0$, while $q_s = w$.

Where, RM_{FB} is the regular margin including the environmental damage without the need for a tax τ_1 and an EPR fee τ_2, and PD_{FB} is similar to the index of product differentiation, but before the public policies economic incentives and without really representing the substitutability of products.

$$RM_{FB} = \alpha_1 - c - D$$

$$PD_{FB} = \beta_1\beta_2 - 4\gamma^2$$

Where a constraint on $PD_{FB} > 0$ is needed to ensure positive quantities. The first best results are divided in two situations.

When the KKT condition is not saturated ($\lambda = 0$ and $w > q_s$), the expressions of quantities and prices at the social optimum become:

$$w^* = \frac{\alpha_2[\beta_2 RM_{FB} - 2\gamma] - PD_{FB} \times D}{2cPD_{FB} - \alpha_2^2\beta_2} \tag{4.16}$$

$$q_r^* = \frac{2c[\beta_2 RM_{FB} - 2\gamma] - \alpha_2\beta_2 D}{2cPD_{FB} - \alpha_2^2\beta_2} \tag{4.17}$$

$$q_s^* = \frac{\beta_1 - 2\gamma RM_{FB}}{PD_{FB}} - \frac{2\alpha_2\gamma}{2cPD_{FB} - \alpha_2^2\beta_2}\left[\frac{[\beta_2 RM_{FB} - 2\gamma]}{PD_{FB}} - D\right] \tag{4.18}$$

$$\begin{aligned} p_r^* =& \frac{\beta_1\gamma + (\beta_1\beta_2 - 2\gamma^2)(c + D)}{PD_{FB}} - \frac{2\gamma^2}{PD_{FB}}\left[\alpha_1 + \frac{\alpha_2^2[\beta_2 RM_{FB} - 2\gamma]}{2cPD_{FB} - \alpha_2^2\beta_2}\right] \\ &+ \frac{2\alpha_2\gamma^2 D}{2c_1 PD_{FB} - \alpha_2^2\beta_2} \end{aligned} \tag{4.19}$$

$$\begin{aligned} p_s^* =& 1 + \frac{\beta_2\gamma}{2cPD_{FB} - \alpha_2^2\beta_2}\left[2cRM_{FB} - \frac{\alpha_2}{2c - \alpha_2^2\beta_2}\left(\frac{2\alpha_2\gamma}{PD_{FB}} + D\right)\right] \\ &- \frac{(\beta_1\beta_2 - 2\gamma^2)}{PD_{FB}} \end{aligned} \tag{4.20}$$

When the KKT condition is saturated ($\lambda > 0$ and $q_s = w$), the expressions of quantites and prices at the social optimum become:

$$w^* = q_s^* = \frac{\beta_1(1 - D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2} \tag{4.21}$$

$$q_r^* = \frac{\alpha_1 - c - D}{\beta_1} + \frac{(\alpha_2 - 2\gamma)}{\beta_1}\left[\frac{\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2}\right] \tag{4.22}$$

$$p_r^* = \frac{\gamma[\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)]}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2} + c + D \tag{4.23}$$

$$p_s^* = 1 - \frac{\gamma(\alpha_1 - c - D)}{\beta_1} - [\gamma(\alpha_2 - 2\gamma)]\left[\frac{\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2}\right] \tag{4.24}$$

Proposition 2 *The expressions of quantities and prices at the public optimum are defined by equations (4.16) to (4.20) when the optimum allows for unsold inventories, and by equations (4.21) to (4.24) in the absence of unsold inventories.*

Difference with the Retailer's Producer Problem When comparing the private optimal outcome, equations (4.9 to 4.13) to the social optimal values when unsold inventories are allowed, equations (4.16 to 4.20), we see that the *first best* equilibrium results have similar but different quantities than the ones in the retailer's problem.

Initially, the comparison between the retailer equilibrium and *first best* quantities is done by holding the public policies null ($\tau_1 = 0$ and τ_2=0). This shows a comparison without the intervention of a social planner.

Table 4.1 Quantities Comparison with $\tau_1 = 0$ and $\tau_2 = 0$

Retailer Equilibrium		First Best
$\bar{w}$	$>$	w^*
$\bar{q}_r$	$>$	q_r^*
$\bar{q}_s$	$<$	q_s^*

For the quantities brought to market ($w + q_r$) it shows the expected difference where the retailer equilibrium displays larger quantities than the *first best*. However, the quantity sold on sale is larger for a social optimum.

Then, the comparison between the quantities related to trends $\bar{w}$ and w^* opposes a decrease of the denominator to a proportional increase of the numerator. However, for both expressions to be equal, the values of the tax and fee such as $\tau_1 = 1$ and $\tau_2 = D$. The results of fixing w is as follow.

Table 4.2 Quantities Comparison with $\tau_1 = 1$ and $\tau_2 = D$

Retailer Equilibrium		First Best
$\bar{w}$	$=$	w^*
$\bar{q_r}$	$<$	q_r^*
$\bar{q_s}$	$<$	q_s^*

Looking at the difference between both quantities sold at regular price q_r, there is the a similar change with an increase of the numerator and a decrease of the denominator. However, the denominator decreases more than the numerator increases. Hence, the quantity decided by the retailer $\bar{q_r}$ is smaller than the *first best*. For the quantities sold on sale q_s, the difference follows a similar comparison. This holds even when $\tau_1 = 1$ and $\tau_2 = D$.

By choosing taxes so that the decentralized quantity related to trend is equal to the socially optimal one $\bar{w} = w^*$, we see that the retailer will not sell enough quantities $\bar{q_r} + q_s$. In other words, private production fails to generate enough welfare, while generating too much unsold inventories.

Both comparisons show how a *first best* is impossible to reach with the use of τ_1 and τ_2 alone and that a focus on quantities related to trends w leads to a lack of q_r and q_s. Moreover, the difference between both equilibrium comes also from the two stage decision-making of the retailer's production problem. While, the social planner decides on all three quantities simultaneously for the social optimum.

4.5 Analysis

Along with the public policies tax τ_1 and fee τ_2, there are parameters introduced in the model requiring further analysis to understand their effect. Notably, consumers' willingness to pay for novelty α_2 and the substitutability, shown by γ, give insights on the changes in quantities decided by the retailer.

As exogenous parameters, a comparative static analysis enables to explore those changes. The method is to take the partial derivative of isolated elements of the function with respect to the studied parameter. The result's sign gives insight on the direction of changes.

The changes on the decision-making of the retailer for their profit function π are shown through the analyses of the quantities found at the equilibrium. While, it is also interesting to analyse the effect of those parameters on the social welfare function SW. However, as the social planner cannot impose the quantities found at the *first best* for the retailer's production, they can introduce public policies to influence the retailer's decision-making.

Then, it is the SW function at 4.15 with the quantities $\bar{w}$, $\bar{q_r}$ and $\bar{q_s}$ found on the retailer's problem.

$$\begin{aligned} SW = &\int_0^{\bar{q_r}} [p_r(q_r) - \bar{p_r}]\, dq_r + \int_0^{\bar{q_s}} [p_s(q_s) - \bar{p_s}]\, dq_s \\ &+ \bar{p_r}\bar{q_r} + \bar{p_s}\bar{q_s} - c(\bar{q_r} + \bar{w}^2) - D(\bar{q_r} + \bar{w}) \end{aligned} \tag{4.25}$$

The quantities from the retailer's equilibrium are a result of inactive KKT conditions, where unsold inventory is present. Hence, the analysis of the effect of public policies is specific for the studied scenario where a retailer needs to dispose of their surplus.

4.5.1 Presence of Public Policies

As the *first best* quantities are unreachable, the social planner introduces public policies that have an effect on the social welfare function. The first analysis concerns the tax τ_1, while the second is for the EPR fee τ_2.

Tax τ_1

The aim of a tax τ_1 for the disposal of unsold inventory is to add a cost to deter the retailer from resorting to landfills and incineration as it is less expensive and more recurring.

Table 4.3 Comparative Static Analysis for τ_1

Social Welfare		
Component	Partial Derivative	Sign
CS_{q_r}	$\frac{\partial CS_{q_r}}{\partial \tau_1}$	< 0
CS_{q_s}	$\frac{\partial CS_{q_s}}{\partial \tau_1}$	> 0
CS	$\frac{\partial CS}{\partial \tau_1}$	$\lesseqgtr 0$
Profits Before Taxes	$\frac{\partial Profits\ Before\ Taxes}{\partial \tau_1}$	< 0
Externalities	$\frac{\partial Externalities}{\partial \tau_1}$	< 0
Retailer Equilibrium		
Quantity	Partial Derivative	Sign
$\bar{w}$	$\frac{d\bar{w}}{d\tau_1}$	< 0
$\bar{q}_r$	$\frac{d\bar{q}_r}{d\tau_1}$	< 0
$\bar{q}_s$	$\frac{d\bar{q}_s}{d\tau_1}$	> 0
$(\bar{w} - \bar{q}_s)$	$\frac{d(\bar{w}-\bar{q}_s)}{d\tau_1}$	< 0

As a tax, the direct effect expected is a reduction of unsold inventory, as seen on 4.3. This change comes from decreasing the total quantity produced $(\bar{w}+\bar{q}_r)$ and increasing the quantity sold on sale $\bar{q}_s$. Since the unsold inventory is represented by $(q_s - w)$, the boost of q_s is meant to minimize the unsold inventory in need of disposal at the end of the selling season. This adjustment benefits the consumer surplus from purchasing apparel on sale. However, it is ambiguous how the overall consumer surplus behaves with a decrease of CS_{q_r} and an increase of CS_{q_s}.

The reduction of the total quantity produced happens because an additional cost is added to the portion of production that might remain unsold. The contraction of total production softens the production costs c and the related environmental damage. However, the consumer surplus from regular price apparel diminishes due to a lower quantity available at regular price $\bar{q}_r$, which reduces the profits before taxes.

EPR fee τ_2

While the tax on disposal τ_1 is applied to unsold inventory, an EPR fee τ_2 concerns the total quantity brought to market $(\bar{q}_r + \bar{w})$. Hence, instead of being a cost to a specific portion of produced quantities, an EPR fee adds on the production costs as it aims to internalize the end of life environmental impact, whether or not the product has been consumed.

Table 4.4 Comparative Static Analysis for τ_2

Social Welfare		
Component	**Partial Derivative**	**Sign**
CS_{q_r}	$\frac{\partial CS_{q_r}}{\partial \tau_2}$	< 0
CS_{q_s}	$\frac{\partial CS_{q_s}}{\partial \tau_2}$	> 0
CS	$\frac{\partial CS}{\partial \tau_2}$	$\lesseqgtr 0$
Profits Before Taxes [2]	$\frac{\partial Profits\ Before\ Taxes}{\partial \tau_2}$	< 0
Externalities	$\frac{\partial Externalities}{\partial \tau_2}$	< 0
Retailer Equilibrium		
Quantity	**Partial Derivative**	**Sign**
$\bar{w}$	$\frac{d\bar{w}}{d\tau_2}$	< 0
$\bar{q_r}$	$\frac{d\bar{q_r}}{d\tau_2}$	< 0
$\bar{q_s}$	$\frac{d\bar{q_s}}{d\tau_2}$	> 0
$(\bar{w} - \bar{q_s})$	$\frac{d(\bar{w}-\bar{q_s})}{d\tau_2}$	< 0

Introducing an EPR-type legislature that increase the costs of bringing products to the market reduces the total costs of production and environmental damage due to an overall decrease of the total produced quantity $(\bar{q_r} + \bar{w})$. However, while it increases the consumer surplus from purchasing products on sale, it decreases purchases of products at regular price.

Similarly to the tax on disposal τ_1, there is an evident decrease of $\bar{w}$ with an increase of $\bar{q_s}$ in order to reduce the unsold inventory $(\bar{w} - \bar{q_s})$.

The effect of both economic incentives introduced by the public policies is to reduce the total produced quantity $(\bar{q_r} + \bar{w})$, which limits the environmental impact.

Proposition 3 *The introduction of a tax τ_1 and an EPR fee τ_2 has both the same effect on the quantities variables of the retailer equilibrium:*

- *Total quantity produced $(\bar{w} + \bar{q_r})$ decreases,*

[2] A condition arises from $\frac{dProfits\ Before\ Taxes}{d\tau_2} < 0$, where for it to be respected $\alpha_2 > 2c$. In addition, for the second order condition to be respected, $2c > \frac{\alpha_2^2 \beta_2}{4(\beta_1\beta_2 - \gamma^2)}$. Together, both constraints become $\alpha_2 > 2c > \frac{\alpha_2^2 \beta_2}{4(\beta_1\beta_2 - \gamma^2)}$.

- *Unsold inventory* $(\bar{w} - \bar{q_s})$ *decreases,*
- *Profits before taxes decrease,*
- *Environmental damages decrease,*
- *Ambiguous effect on consumers surplus.*

4.5.2 Willingness to Pay for Novelty α_2

As fast fashion accelerates trends cycles, it pushes consumers to an increasing desire for novelty. This willingness to pay is represented by the parameter α_2.

Table 4.5 Comparative Static Analysis for α_2

Social Welfare		
Component	Partial Derivative	Sign
CS_{q_r}	$\frac{\partial CS_{q_r}}{\partial \alpha_2}$	< 0
CS_{q_s}	$\frac{\partial CS_{q_s}}{\partial \alpha_2}$	> 0
CS	$\frac{\partial CS}{\partial \alpha_2}$	$\lesseqgtr 0$
Profits Before Taxes	$\frac{\partial Profits\,Before\,Taxes}{\partial \alpha_2}$	< 0
Externalities	$\frac{\partial Externalities}{\partial \alpha_2}$	< 0
Retailer Equilibrium		
Quantities	Partial Derivative	Sign
$\bar{w}$	$\frac{d\bar{w}}{d\alpha_2}$	> 0
$\bar{q_r}$	$\frac{d\bar{q_r}}{d\alpha_2}$	< 0 [3]
$\bar{q_s}$	$\frac{d\bar{q_s}}{d\alpha_2}$	> 0
$(\bar{w} - \bar{q_s})$	$\frac{d(\bar{w}-\bar{q_s})}{d\alpha_2}$	> 0

The effect of consumers willingness to pay for novelty α_2 is a positive influence on the quantity related to trends $\bar{w}$. Indeed, if consumers value novelty more, then retailers will bring more of new collections to market. With an increase of $\bar{w}$, there is more products being sold on sale $\bar{q_s}$. Hence, there is a preference for selling on sale and a reduction of the quantity sold at regular price $\bar{q_r}$.

[3] Due to $(8c(\beta_1\beta_2 - \gamma^2) + \alpha_2^2\beta_2)(\tau_1 + \tau_2) > 4c\alpha_2[\beta_2(\alpha_1 - c - \tau_2) - \gamma(1 + \tau_1)]$.

Importantly, an increase of the parameter α_2 has a negative impact on profits before taxes, while the overall effect of opposing consumer surplus is ambiguous. If the sum of consumer surplus decreases when α_2 increases, then it is a scenario where the only positive impact is the decrease[4] of environmental damage due to a lower volume of production. This situation indicates how a focus on trends pushes an increase in overall prices for solely selling the fashionable collections. Hence, consumers bear the burden of a costly system that does not benefit them in terms of purchase choices.

Proposition 4 *The increase of consumers willingness to pay for novelty has an effect on the quantities variables of the retailer equilibrium:*

- *Total quantity produced* $(\bar{w} + \bar{q_r})$ *increases,*
- *Unsold inventory* $(\bar{w} - \bar{q_s})$ *increases,*
- *Profits before taxes increase,*
- *Environmental damages increase,*
- *Ambiguous effect on consumers surplus.*

4.5.3 Substitutability

The parameter γ explores the elasticity between the products at regular price and on sale. While clothing at regular price is sold at the start of the season when the trend is new, clothing sold on sale is the same product, but at a moment where the trend is fading. So, the parameter γ examines the substitutability between the same product at these two moments.

When the products are complementary ($PR_P = \beta_1\beta_2$), γ is 0 , resulting in two consumer types: 1) a customer that highly values to wear the trend at the start of the season, and 2) a customer who only buys on sale items. For them, there is no substitution.

[4]It comes from $D(\frac{\partial \bar{q_r}}{\partial \alpha_2} + \frac{\partial \bar{w}}{\partial \alpha_2}) < 0$.

Table 4.6 Comparative Static Analysis for γ

Social Welfare		
Component	Partial Derivative	Sign
CS_{q_r}	$\frac{\partial CS_{q_r}}{\partial \gamma}$	< 0
CS_{q_s}	$\frac{\partial CS_{q_s}}{\partial \gamma}$	< 0
CS	$\frac{\partial CS}{\partial \gamma}$	< 0
Profits Before Taxes	$\frac{\partial Profits\, Before\, Taxes}{\partial \gamma}$	> 0
Externalities	$\frac{\partial Externalities}{\partial \gamma}$	< 0
Retailer Equilibrium		
Quantities	Partial Derivative	Sign
$\bar{w}$	$\frac{d\bar{w}}{d\gamma}$	> 0
$\bar{q}_r$	$\frac{d\bar{q}_r}{d\gamma}$	> 0
$\bar{q}_s$	$\frac{d\bar{q}_s}{d\gamma}$	< 0
$(\bar{w} - \bar{q}_s)$	$\frac{d(\bar{w}-\bar{q}_s)}{d\gamma}$	> 0

With an increase of γ, a substitution is introduced, which influences a fast fashion retailer decision-making as visible in Table 4.6. The effect is similar for both the retailer's production problem and for the social planner optimizing the social welfare function: the quantity related to trends w increases and the quantities sold at regular price q_r and on sale q_s decrease.

Proposition 5 *The increase of substitutability between new clothing and end-of-the season has an effect on the quantities variables of the retailer equilibrium:*

- *Total quantity produced* $(\bar{w} + \bar{q}_r)$ *increases,*
- *Unsold inventory* $(\bar{w} - \bar{q}_s)$ *increases,*
- *Profits before taxes increase,*
- *Environmental damages decreases,*
- *Consumers surplus decreases.*

4.6 Discussion

4.6.1 Return on Results

Retailer Equilibrium

The analysis and comparison are done with the results of two optimization problems: the retailer equilibrium and the *first best* social welfare. The retailer's production problem results in three quantity variables $\bar{w}$, $\bar{q_r}$ and $\bar{q_s}$, while the social planner decides on other quantities for the social optimum w^*, q_r^* and q_s^*. The retailer optimizes their profit function, which incorporates two public policies (a tax on disposal τ_1 and an EPR fee τ_2) and the inverse demand function that includes the consumers willingness to pay for novelty α_2 and the cross-elasticity between the product at regular price at the start of the season and the same product sold on sale at the end γ.

As economic incentives that add costs to production, the public policies τ_1 and τ_2 have the expected effect of decreasing the total produced quantity $(\bar{q_r}+\bar{w})$. It translates in a reduction of profits for the retailer and an ambiguous effect on consumers surplus, but with a positive reduction of environmental impact.

For the parameters of novelty α_2 and substitutability γ, their effects on the retailer's decision-making are similar: an increase of quantities related to trends $\bar{w}$ and sold on sale $\bar{q_s}$, while the quantity sold at regular price q_r decreases. It shows an ambiguity where, on one side, an increase in novelty puts pressure on retailers to bring fashionable collections to stores. On the other hand, an increase in substitutability has the same effect, while it means less interest to buy at the start of the season, but still valuing newness.

Social Welfare

The introduction of an additional cost τ_1 for the disposal of unsold inventory has the expected effect of a tax. Indeed, τ_1 reduces the overall consumer surplus by reducing the welfare from the purchase of a product on regular price and on sale. It is a result from the reduction of the overall quantity produced. The benefit from it is less environmental externalities from production, which is the main source of environmental impact in the fashion industry. Therefore, such an additional cost has the targeted effect of limiting the production of clothing that will, ultimately, never be worn. Similarly, an EPR fee τ_2 adds a cost on the production of everything the retailer brings to market. As with the tax, the overall consumer surplus decreases, while improving the environmental impact.

4.6.2 Strength and Limits

A strength of the model introduced in this article is that it considers the effect of the willingness to pay for fashionable novelty by consumers on the decision-making process of fast fashion retailers optimizing their production. This addition is important to consider the real purchase behaviour of customers in the fashion industry as shown in the literature [27]. This parameter for novelty α_2 and its effect on production through the quantity related to trends w are an original contribution to the literature on fast fashion. Differently than the newsvendor problem approach in the literature, it addresses the specific relation between consumers desire for newness manufactured by trend cycles and the opportunity that retailers seize to make a profit.

The social welfare function is part of an approach that internalises negative environmental impact. However, the global context of fast fashion makes it so that the significant environmental damage occurs in producer regions, while the benefit of consumption happens in the main importing countries. For a social planner to impose the regulations introduced in the model, it would need to be a policy globally applied. It is reminiscent of the polluter's paradise concept, where countries with low environmental regulations attract polluting producing operations for a consumption overseas.

While the analysis looks at the substitutability, it fails to determine if $(w + q_r)$ increases overall with the parameter γ. This ambiguity could be further addressed.

4.6.3 Perspectives

The tax τ_1 and the fee τ_2 respectively represent a tax on the disposal of unsold inventory and an EPR fee for products to market. Both are promising public policies with examples of application worldwide (e.g. France's EPR on textiles). However, even if Canada is among the consumer countries, its context is distinct. Notably, an EPR-type legislation has been planned in a Canadian strategy, but ultimately it is still not applied. Hence, the application of such regulations needs a better an clearer analysis of the mechanisms and dynamics of stakeholders present in Canada.

4.7 Conclusion

This article proposes an original optimization model for a retailer in the fashion industry facing the challenge of disposing of unsold inventory. Mainly, these never worn apparel are sent to landfills worldwide due to a lack of viable alternatives for textile waste treatments.

Due to a fast fashion context, this issue is exacerbated by fast trend cycles, products' overall lower quality and consumers desire newness.

The model considers the effect of this willingness to pay for novelty on the retailer's decision-making while introducing public policies to deter retailers from destroying their excess inventory and internalize their negative externalities. Other than novelty, the construction of the model includes the substitutability between clothing at the start of the trend and at the end, and the introduction of economic incentives from public policies. Then, it proceeds with the solution of the retailer's production problem that results in an equilibrium, before finding the social optimum through a social planner maximizing the social welfare. The analysis of both equilibrium shows the potential of a tax on disposal and an EPR fee to reduce the environmental impact of the industry by decreasing production, and gives an essential insight on the ambiguity of their effect on social welfare.

Overall, this article findings require further analysis to better understand available solutions to the issue of deadstock destruction by retailers.

CHAPTER 5 GENERAL DISCUSSION

5.1 Return on the Article on Chapter 4

5.1.1 Results

The article on Chapter 4 results on two different equilibrium: the retailer's production problem and the social optimum. The first equilibrium derives from the decisions of a monopolistic retailer optimizing their profits in a sequence of two choices. For the first decision, they optimize the quantity brought to market which are clothing collections that follow the latest trends w. Then, the retailer proceeds with the decision on the total quantity brought to market by deciding on the quantity sold at regular price q_r and the quantity sold on sale q_s. While the second equilibrium derives from a social planner that decides simultaneously on these three quantity variables that the monopolistic retailer should bring to market and sell in order to maximize a social welfare function.

Public Policies

The introduction of both public policies has the effect of a negative economic incentive as it reduces the total produce quantity $(\bar{q_r} + \bar{w})$. It translates in a reduction of profits for the retailer and an ambiguous effect on consumers surplus. As a additional costs, it is expected that the overall effect on consumers surplus is a reduction as well. Then, the environmental impact of production is reduced due to less products on the market.

Novelty

The effect of the willingness to pay for novelty α_2 has a direct positive effect on the production of a collection of clothing that follows the fashionable trend. Also, it has a double effect on the quantities actually sold to customers. An increase of desire for novelty from consumers presses the retailer to increase the supply of trendy apparels w. This leads to higher prices at the beginning of the season $\bar{p_r}$, with the consequence of a reduced quantity to be sold at regular price $\bar{q_r}$, but an increased quantity to be sold on sale $\bar{q_s}$. It represents well the pressure that retailers have to fulfill customers' desire for new trends manufactured by fast fashion. While the total impact on consumer surplus stays ambiguous, profit before taxes is reduced and externalities increase.

Substitutability

The parameter γ introduces the expected effect of cross-elasticity for the clothing sold at regular price and on sale. The result on the total produced quantity $(w + q_r)$ is ambiguous due to the increase of w and the decrease of q_r. However, it is expected that when the products are perfect substitutes $(PR = 0)$, the quantity sold on sale q_s reaches its maximum value where it equals w. At that point, there is no unsold inventory $(w - q_s)$ leftover after the selling season. Hence, when the newness effect of fast fashion on consumers is reduced, the disposal of surplus issue is avoided. So, with more substitution, there is less trend cycles and a reduced environmental impact from production and end-of-life disposal.

Social Welfare

The introduction of an additional cost τ_1 for the disposal of unsold inventory has the expected effect of a tax. Indeed, τ_1 reduces the overall consumer surplus by reducing the welfare from the purchase of a product on regular price and on sale. It is a result from the reduction of the overall quantity produced. The benefit from it is less environmental externalities from production, which is the main source of environmental impact in the fashion industry. Therefore, such an additional cost has the targeted effect of limiting the production of clothing that will, ultimately, never be worn. Similarly, an EPR fee τ_2 adds a cost on the production of everything the retailer brings to market. As with the tax, the overall consumer surplus decreases, while improving the environmental impact.

5.2 Strength and Limits

Overall, this communication explores the link between fast fashion garment production and textile waste issues. The linearity of the industry ensures that an increase in production will worsen garments' end-of-life environmental impact. Sustainability in fashion is more often approached through design and material choices than through economic lenses. In particular, the destruction of unsold inventory exemplifies how a firm's decision-making in the current industry state and regulations leads to inefficient sustainable choices. Furthermore, this overview discussed the known causes of a garment's environmental impact, how little is known for the end-of-life phase and some reasons why.

A strength of the model introduced in this article is that it considers the effect of the willingness to pay for fashionable novelty by consumers on the decision-making process of fast fashion retailers optimizing their production. This addition is important to consider the real purchase behaviour of customers in the fashion industry as shown in the literature [27]. The parameter for novelty α_2 and its effect on quantities related to trends w that a retailer decides to bring to market are an original contribution to the literature on fast fashion. It provides a different approach to understand the specific issue of waste in the industry and adds insight to the knowledge surrounding the effects of fast fashion.

The social welfare function is part of an approach that internalises negative environmental impact. However, the global context of fast fashion makes it so that the significant environmental damage occurs in producer regions, while the benefit of consumption happens in the main importing countries. For a social planner to impose the regulations introduced in the model, it would need to be a policy globally applied. It is reminiscent of the polluter's paradise concept, where countries with low environmental regulations attract polluting producing operations for a consumption overseas.

While the analysis looks at the substitutability, it fails to determine if $w + q_r$ increases overall with the parameter γ. This ambiguity could be further addressed.

For simplicity, the model represents a conventional retailer with street-side stores. It allows an initial analysis of the parameters and public policies that gives an insight of their effects considering the influence of fast fashion on purchase behaviors and on production. However, the distribution and selling strategies by retailers is more complex in this same context of fast fashion. Mainly, fast fashion retailers and brands have developed a significant online presence and omnichannel distribution strategy. These would complexify the model, but their expected effect would be to increase profits as they aim to facilitate purchases.

5.3 Perspectives

The digitisation of the fashion industry goes beyond retailers strategies, as it spreads to parts of the supply chain that lack a direct contact with the public. It would be crucial to take the digitisation of the fashion industry into account. Often as an operational and logistical tool, it will optimize resources efficiency in production steps. The growing adoption of these tools by manufacturers is expected to increase sales as it increases productivity. Even if this efficiency tends to prevent waste in the supply chain, the growth of production can have a negative environmental impact trough overproduction.

The article on Chapter 4 highlights the difference between the quantities produced and sold by a monopolistic retailer's decision-making and the quantities of the social optimum found by a social planner. The comparison between these two equilibrium highlights the impossibility of reaching the *first best* quantities. Which leads to a need to find the *second best* solution with exact expressions for the tax on disposal τ_1 and the EPR fee τ_2.

The tax τ_1 and the fee τ_2 respectively represent a tax on the disposal of unsold inventory and an EPR fee for products to market. Both are promising public policies with examples of application worldwide (e.g. France's EPR on textiles). However, even if Canada is among the consumer countries, its context is distinct. Notably, an EPR-type legislation has been planned in a Canadian strategy, but ultimately it is still not applied. Hence, the application of such regulations needs a better an clearer analysis of the mechanisms and dynamics of stakeholders present in Canada. These are key insights for Canadian policymakers, as it is a crucial knowledge for the catching up of federal and provincial regulations compared to the countries in the forefront of the fight against the impacts of fast fashion. The solutions that governments must put in place are broader than environmental policies, as retail and international trade regulations also hold a potential to steer a systemic transition.

This document contributes to a field in need of attention from researchers and policymakers, since the gap to fill in available data and knowledge of stakeholders is significant. Specially, it is one of a handful of works done that considers the Canadian context and economic aspects of the industry. Yet, transitioning fashion towards more circular business models is crucial to change the environmental impact from the global consumer goods industry.

CHAPTER 6 CONCLUSION AND RECOMMENDATIONS

6.1 Summary of Works

6.1.1 Literature Review

The literature review on Chapter 2 allowed to explore the existing situation surrounding the unsold inventory destruction practice of brands and retailers. Mainly, it looks into detail on the aspects of fast fashion, its effect on the industry's environmental impact and how it is a major cause of textile waste.

Then, the review brings out how the linearity of the fashion industry and it's globalized context create an imbalance of the environmental impact in countries where the production happens, while sparing "consumer" countries. This highlights the complexity of environmental issues around fashion, but also the importance of considering socioeconomic aspects.

Instead of the strategies available to retailers to reduce their impact, from ecodesign to production processes, this research project focused on economic incentives implemented by public policies. Governmental action is at the center of solutions for a more sustainable fashion industry as it can directly and indirectly influence stakeholders.

The unrestrained destruction of unsold inventory worsens the textile waste management issues of the industry by increasing the quantity of clothing to be treated. Public policies are key tools to uphold the rise of alternative disposal methods or restrict impactful practices. Subsidies, or taxes, can help finance recycling operations and assist in covering the costs of barriers to larger scale operations. Whereas regulations on riskier materials and processes can limit the scale of their environmental impact. Similarly, constraints on unsold products' destruction can hinder, if not ban, the possibility of using landfills for never worn clothing. The French example of requiring retailers to keep their products in a second-hand market is of particular interest as firms are obligated to contribute to the financing of disposal systems designed to recover at least part of the product's value. France, Netherlands, Sweden and the European Union display a global leadership regarding support of alternatives addressing the issues of textile waste management. Canada shares the "consumer" status of those countries, but federal and provincial initiatives are still to be implemented. As Canadian policymakers took similar commitments towards the UN Sustainable Development Goals and the Paris Agreement as did these global leaders, there is a need to adopt a similar attitude.

6.1.2 Article

Fast fashion exacerbates the industry's environmental impact by speeding trend cycles and lowering product's quality. For unsold inventory at the end of a season, the clothing looses its value due to the fading of the trend. Retailers dispose of excess stock through landfills and incineration as alternatives are limited (reuse is a saturated market and recycling, is in early development). Hence, the article proposes economic incentives introduced by public policies to inhibit unsold inventory destruction.

Considering fast fashion's effect on manufactured desire for newness through quick trend cycles, there was a need to construct an optimization model that includes consumers' willingness to pay for novelty. The model also explores the influence of substitutability of fashionable clothing at regular price with the same product on sale at the end of the trend.

Through the comparison of a monopolistic retailer's production problem and the social optimum, the article highlights the need for public policies that influence the retailer's decision-making. Both public policies introduced have an effect of reducing the total produced quantity. However, as they reduce consumption, they also reduce the environmental impact. This is an ambiguous trade-off for the social welfare that needs further examination.

6.2 Future Research

The original optimization model developed on Chapter 4 contributes insights on the effects of fast fashion on a retailer's production problem and social welfare by taking into account consumers' desire for newness. To expand on the model findings and improve it as an accessory for policymaking, there are more aspects to consider: an additional optimization problem and different strategies available to retailers to reduce their externalities.

In terms of optimization, the model highlights the need to further detail the expression of a tax on disposal τ_1 and an EPR fee τ_2 through the development of a *second best* equilibrium. This is a first improvement to advance the insights from the model and explore the capability of chosen public policies to inhibit the destruction of unsold inventory. Then, there are other suitable public policies that lean on regulations and on transferring information to consumers instead of introducing economic incentives.

On the other hand, retailers have a range of strategies available to reduce their externalities. Still on the production side there are ecodesign choices, more sustainable materials and low-impact processes. Then, as textile recycling grows, it would be necessary to understand how it would change the findings of the model. These are all additional elements present on the literature review and that can improve the insights from the optimization model.

As sustainability in fashion increases as an issue, the fate of textile waste should be a focus for researchers, firms, and policymakers. Reduction of the environmental impact related to our consumption of clothing is linked to an ecosystem of interconnected solutions and a decline of damageable practices such as the destruction of unsold products. Hence, there is a critical need to understand the application of proposed solutions and what outcomes to expect.

REFERENCES

[1] Fashion Revolution. (2021) Written evidence submitted by fashion revolution. [Online]. Available: http://data.parliament.uk/writtenevidence/committeeevidence.svc/evidencedocument/environmental-audit-committee/sustainability-of-the-fashion-industry/written/90117.html

[2] K. Fletcher, *Sustainable Fashion and Textiles: Design Journeys*, 2nd ed. London, UK: Earthscan from Routledge, 2014.

[3] H. of Commons Environmental Audit Committee, "Fixing Fashion : clothing consumption and sustainability," February 2019.

[4] Grand View Research. Textile market size, share and trends analysis report by raw material (wool, chemical, silk, cotton), by product (natural fibers, polyester, nylon), by application, by region, and segment forecasts, 2020 - 2027. [Online]. Available: https://www.grandviewresearch.com/industry-analysis/textile-market

[5] Textile Exchange, "Preferred Fiber and Materials Market Report 2019," pp. 1–87, 2019.

[6] ——, "Preferred Fiber and Materials Market Report 2020," p. 103, 2020. [Online]. Available: https://textileexchange.org/wp-content/uploads/2020/06/Textile-Exchange_Preferred-Fiber-Material-Market-Report_2020.pdf

[7] J. Kerr and J. Landry, "Pulse of the Fashion Industry," *Global Fashion Agenda and The Boston Consulting Group*, 2017. [Online]. Available: http://globalfashionagenda.com/wp-content/uploads/2017/05/Pulse-of-the-Fashion-Industry_2017.pdf

[8] M. Bruce, L. Daly, and N. Towers, "Lean or agile: A solution for supply chain management in the textiles and clothing industry?" *International Journal of Operations and Production Management*, vol. 24, no. 1-2, pp. 151–170, 2004.

[9] K. O'Marah, "A Reality Check on the Collaboration Dreams," no. 1, pp. 0–6, 2021.

[10] L. Macchion *et al.*, "Strategic approaches to sustainability in fashion supply chain management," *Production Planning and Control*, vol. 29, no. 1, pp. 9–28, 2018. [Online]. Available: http://doi.org/10.1080/09537287.2017.1374485

[11] P. Rivoli, *The travels of a t-shirt in the global economy: An economist examines the markets, power, and politics of world trade. New preface and epilogue with updates on economic issues and main characters.* John Wiley & Sons, 2014.

[12] M. Mehrjoo and Z. J. Pasek, "Impact of product variety on supply chain in fast fashion apparel industry," *Procedia CIRP*, vol. 17, pp. 296–301, 2014. [Online]. Available: http://dx.doi.org/10.1016/j.procir.2014.01.082

[13] G. P. Cachon and R. Swinney, "The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior," *Management Science*, vol. 57, no. 4, pp. 778–795, 2011.

[14] A. Sen, "The US fashion industry: A supply chain review," *International Journal of Production Economics*, vol. 114, no. 2, pp. 571–593, 2008.

[15] L. Barnes and G. Lea-greenwood, "Fast fashioning the supply chain : shaping the research agenda," vol. 10, no. 3, pp. 259–271, 2006.

[16] Ellen MacArthur Foundation, "A new textiles economy: Redesigning fashion's future," Tech. Rep., 2017. [Online]. Available: https://www.ellenmacarthurfoundation.org/publications/a-new-textiles-economy-redesigning-fashions-future

[17] R. Bick, E. Halsey, and C. C. Ekenga, "The global environmental injustice of fast fashion," *Environmental Health: A Global Access Science Source*, vol. 17, no. 1, pp. 1–4, 2018.

[18] M. S. Andersen, "An introductory note on the environmental economics of the circular economy," *Sustainability Science*, vol. 2, no. 1, pp. 133–140, 2007.

[19] S. Roos *et al.*, "A life cycle assessment (LCA)-based approach to guiding an industry sector towards sustainability: the case of the Swedish apparel sector," *Journal of Cleaner Production*, vol. 133, pp. 691–700, 2016.

[20] K. Niinimaki *et al.*, "The environmental price of fast fashion," *Nature Reviews Earth and Evnironment*, vol. 1, 2020.

[21] Quantis, "Measuring fashion 2018: Environmental Impact of the Global Apparel and Footwear Industries Study Full report and methodological considerations," p. 65, 2018. [Online]. Available: https://quantis-intl.com/measuring-fashion-report-2018/

[22] R. Kant, "Textile dyeing industry an environmental hazard," *Natural Science*, vol. 04, pp. 22–26, 2012.

[23] N. Kelderman, "Increasing the use of recycled textiles through business models and policies. A study providing practical insights based on industry front-runners," no. March, p. 100, 2019.

[24] V. Bhardwaj and A. Fairhurst, "Fast fashion: Response to changes in the fashion industry," *International Review of Retail, Distribution and Consumer Research*, vol. 20, no. 1, pp. 165–173, 2010.

[25] L. Mcneill and R. Moore, "Sustainable fashion consumption and the fast fashion conundrum: Fashionable consumers and attitudes to sustainability in clothing choice," *International Journal of Consumer Studies*, vol. 39, no. 3, pp. 212–222, 2015.

[26] F. Jia *et al.*, "The circular economy in the textile and apparel industry: A systematic literature review," *Journal of Cleaner Production*, vol. 259, 2020. [Online]. Available: https://doi.org/10.1016/j.jclepro.2020.120728

[27] M. Christopher, R. Lowson, and H. Peck, "Creating agile supply chains in the fashion industry," *International Journal of Retail and Distribution Management*, vol. 32, no. 8, pp. 367–376, 2004.

[28] E. Napier and F. Sanguineti, "Fashion Merchandisers' Slash and Burn Dilemma: A Consequence of Over Production and Excessive Waste?" *Rutgers Business Review*, vol. 3, no. 2, pp. 159–174, 2018.

[29] Fashion Revolution. (2020) The impact of covid-19 on the people who make our clothes. [Online]. Available: https://www.fashionrevolution.org/the-impact-of-covid-19-on-the-people-who-make-our-clothes/

[30] *How Coronavirus' Effect on the Fashion Industry Reveals Flaws in the Global Economy*, Mar. 2020. [Online]. Available: https://time.com/5794928/coronavirus-fashion-economy/

[31] S. Kumar and P. Malegeant, "Strategic alliance in a closed-loop supply chain, a case of manufacturer and eco-non-profit organization," *Technovation*, vol. 26, no. 10, pp. 1127–1135, 2006.

[32] V. Jacometti, "Circular Economy and Waste in the Fashion Industry," *Laws*, vol. 8, no. 4, p. 27, 2019.

[33] F. Binet *et al.*, *Fast Fashion and Sustainable Consumption.* Springer Singapore, 2019. [Online]. Available: http://dx.doi.org/10.1007/978-981-13-1268-7_2

[34] E. Nathalie Remy, Speelman and S. Swartz. (2016, Oct.) Style that's sustainable: A new fast-fashion formula. [Online]. Available: https://www.mckinsey.com/business-functions/sustainability/our-insights/style-thats-sustainable-a-new-fast-fashion-formula

[35] International Trade Center. Trade map - canada. [Online]. Available: https://www.trademap.org/canada/index.aspx

[36] G. P. Cachon and R. Swinney, "Purchasing, pricing, and quick response in the presence of strategic consumers," *Management Science*, vol. 55, no. 3, pp. 497–511, 2009.

[37] Statistics Canada. Table 20-10-0008-02 retail trade sales by industry (x 1,000). [Online]. Available: https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=2010000802

[38] ——. Table 36-10-0434-04 gross domestic product (gdp) at basic prices, by industry, monthly, industry detail (x 1,000,000). [Online]. Available: https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=3610043404

[39] Y. Qin *et al.*, "The newsvendor problem: Review and directions for future research," *European Journal of Operational Research*, vol. 213, no. 2, pp. 361–374, 2011. [Online]. Available: http://dx.doi.org/10.1016/j.ejor.2010.11.024

[40] G. P. Cachon and M. E. Schweitzer, "Decision Bias in the Newsvendor Problem with a Known Demand Distribution : Experimental Evidence," no. June, 2000.

[41] M. K. Paras, A. Curteza, and G. Varshneya, "Identification of best reverse value chain alternatives: A study of Romanian used clothing industry," *Journal of Fashion Marketing and Management*, vol. 23, no. 3, pp. 396–412, 2019.

[42] S. Cullinane and K. Cullinane, "The Logistics of Online Clothing Returns in Sweden and How to Reduce its Environmental Impact," *Journal of Service Science and Management*, vol. 14, no. 01, pp. 72–95, 2021.

[43] G. Sandin and G. M. Peters, "Environmental impact of textile reuse and recycling – A review," *Journal of Cleaner Production*, vol. 184, pp. 353–365, 2018. [Online]. Available: https://doi.org/10.1016/j.jclepro.2018.02.266

[44] FAO and ICAC, *Measuring sustainability in cotton farming systems*, 2015. [Online]. Available: http://www.fao.org/3/a-i4170e.pdf

[45] A. Rieple and R. Singh, “A value chain analysis of the organic cotton industry: The case of UK retailers and Indian suppliers,” *Ecological Economics*, vol. 69, no. 11, pp. 2292–2302, 2010. [Online]. Available: http://dx.doi.org/10.1016/j.ecolecon.2010.06.025

[46] United Nations Statistics Wiki. (2021, May) Harmonized commodity description and coding systems (hs). [Online]. Available: https://unstats.un.org/wiki/pages/viewpage.action?pageId=87426301

[47] P. Fontell and P. Heikkilä, “Model for circular economy in textiles,” p. 58, 2017.

[48] International Cotton Advisory Committee. Cotton this month, july 2020. [Online]. Available: https://www.icac.org/home/index/

[49] Quantis. Sympatex quantifying the plastic leakage of an outdoor jacket across its life cycle. [Online]. Available: https://quantis-intl.com/casestudy/sympatex-plastic-leakage/

[50] M. K. Paras, R. Pal, and D. Ekwall, “Systematic literature review to develop a conceptual framework for a reuse-based clothing value chain,” *International Review of Retail, Distribution and Consumer Research*, vol. 28, no. 3, pp. 231–258, 2018. [Online]. Available: http://doi.org/10.1080/09593969.2017.1380066

[51] K. Laitala, “Consumers’ clothing disposal behaviour - a synthesis of research results,” *International Journal of Consumer Studies*, vol. 38, no. 5, pp. 444–457, 2014.

[52] UNFCCC, “Fashion Industry Charter for Climate Action,” p. 7, 2018. [Online]. Available: https://unfccc.int/climate-action/sectoral-engagement/global-climate-action-in-fashion/fashion-industry-charter-for-climate-action

[53] Refashion, “French Law on Fighting Waste and on the Circular the Essentials for the CLF industry,” no. March, 2020.

[54] Swedish Government Inquiry, “Skatt på modet – för att få bort skadliga kemikalier.” [Online]. Available: https://www.regeringen.se/49597f/contentassets/5184c042af094e7281343c2b8d31a68e/skatt-pa-modet--for-att-fa-bort-skadliga-kemikalier-sou-202020.pdf

[55] Department of Economic and Social Affairs, United Nations. 12 ensure sustainable consumption and production patterns. [Online]. Available: https://sdgs.un.org/goals/goal12

[56] CCME, “Canada-wide Action Plan for Extended Producer Responsibility - PN 1499,” no. October, 2009.

[57] Government of Canada. Inventory of recycling programs in canada. [Online]. Available: https://www.canada.ca/en/environment-climate-change/services/managing-reducing-waste/overview-extended-producer-responsibility/inventory-recycling-programs.html

[58] CCME, Giroux Environmental Consulting, and CCME, "State of Waste Management in Canada," *Canadian Council of Ministers of Environment*, p. 155, 2014.

[59] B. McDonald, "Extended producer responsibility overview of british columbia," Sep. 2017.

[60] City of Markham, "Textile recycling program strategy - next steps," Nov. 2016.

[61] Retail Council of Canada, "Mémoire pour la consultation publique sur le projet de plan directeur de gestion des matières résiduelles 2020-2025," Jan. 2020. [Online]. Available: http://ville.montreal.qc.ca/portal/page?_pageid=6877,143390419&_dad=portal&_schema=PORTAL

[62] Fashion Takes Action, "A Feasibility Study of Textile Recycling in Canada," no. March, 2021.

[63] T. Hines and M. Bruce, *Fashion marketing: Contemporary issues*, 2007.

[64] S. Berbain *et al.*, "Forecasting , Production and Inventory Management of Short Life-Cycle Products : A Review of the Literature and Case Studies," 2011.

[65] C. X. Wang and S. Webster, "The loss-averse newsvendor problem," vol. 37, pp. 93–105, 2009.

[66] T. Iizuka, "An empirical analysis of planned obsolescence," *Journal of Economics and Management Strategy*, vol. 16, no. 1, pp. 191–226, 2007.

[67] A. Burke, "The Dynamics of Product Differentiation in the British Record Industry," *Journal of Cultural Economis*, vol. 20, no. 2, pp. 145–164, 1996.

[68] G. M. Grossman and C. Shapiro, "Informative Advertising with Differentiated Products," *The Review of Economic Studies*, vol. 51, no. 1, p. 63, 1984.

[69] M. Khouja and S. S. Robbins, "Linking advertising and quantity decisions in the single-period inventory model," *International Journal of Production Economics*, vol. 86, no. 2, pp. 93–105, 2003.

[70] B. Han and H. H. Chouinard, "Product quality, advertising intensity and market size," *Economics Letters*, vol. 124, no. 2, pp. 215–218, 2014. [Online]. Available: http://dx.doi.org/10.1016/j.econlet.2014.05.020

[71] I. M. Taplin, "Global commodity chains and fast fashion: How the apparel industry continues to re-invent itself," *Competition and Change*, vol. 18, no. 3, pp. 246–264, 2014.

APPENDIX A RETAILER PRODUCTION PROBLEM

Profit Function

$$\pi_{(q_r,q_s,w)} = p_r q_r + p_s q_s - c(q_r + w^2) - \tau_1(w - q_s) - \tau_2(q_r + w)$$

Where,

- q_r : quantity sold at regular price;
- q_s : quantity sold at discount;
- w : quantity related to trends;
- p_r : regular price;
- p_s : discount price;
- c : production costs;
- τ_1 : tax on unsold inventory destruction
- τ_2 : EPR fee

Inverse Demand

$$p_r = \alpha_1 + \alpha_2 w - \beta_1 q_r - \gamma q_s$$

$$p_s = 1 - \gamma q_r - \beta_2 q_s$$

Constraint

$$q_s \leq w$$

Lagrangian

Due to the inequality constraint, there is a need for the Karush-Kuhn-Tucker conditions.

$$max_{q_r,q_s}\ \pi$$
$$\text{subject to } q_s \leq w,$$

$$\mathcal{L}_{(q_r,q_s,\lambda)} = \pi_{(q_r,q_s)} + \lambda(w - q_s)$$
$$\mathcal{L}_{(q_r,q_s,\lambda)} = p_r q_r + p_s q_s - c(q_r + w^2) - \tau_1(w - q_s) - \tau_2(q_r + w) + \lambda(w - q_s)$$

First Order Condition

$$\frac{d\mathcal{L}_{(q_r,q_s,\lambda)}}{dq_r} = \alpha_1 + \alpha_2 w - 2\beta_1 q_r - 2\gamma q_s - c - \tau_2 = 0$$

$$\frac{d\mathcal{L}_{(q_r,q_s,\lambda)}}{dq_s} = 1 - 2\gamma q_r - 2\beta_2 q_s + \tau_1 - \lambda = 0$$

$$\lambda\frac{d\mathcal{L}_{(q_r,q_s,\lambda)}}{d\lambda} = \lambda(w - q_s) = 0, \lambda \geq 0$$

Replacing $\bar{q_s}$ in the expression $\bar{q_r}$

$$\bar{q_r}(w) = \frac{\beta_2(\alpha_1 + \alpha_2\bar{w} - c - \tau_2) - \gamma(1 + \tau_1 - \bar{\lambda})}{2(\beta_1\beta_2 - \gamma^2)}$$

Replacing q_r in the $\bar{q_s}(w)$ expression

$$\bar{q_s}(w) = \frac{1}{2\beta_2}\Big[1 + \tau_1 - \lambda - \gamma\Big(\frac{\beta_2(\alpha_1 + \alpha_2 w - c - \tau_2) - \gamma(1 + \tau_1 - \lambda)}{(\beta_1\beta_2 - \gamma^2)}\Big)\Big]$$

$$\bar{q_s}(w) = \frac{\beta_1(1 + \tau_1 - \bar{\lambda}) - \gamma(\alpha_1 + \alpha_2\bar{w} - c - \tau_2)}{2(\beta_1\beta_2 - \gamma^2)}$$

Replacing q_r and q_s in p_r and p_s

$$\bar{p_r}(w) = \frac{\alpha_1 + \alpha_2 w + c + \tau_2}{2}$$

$$\bar{p_s}(w) = \frac{1 - \tau_1 + \bar{\lambda}(w)}{2}$$

Optimize Profit Function by Deciding on w

$$\frac{d\pi_w}{dw} = \frac{\partial\pi}{\partial p_r}\frac{\partial\bar{p_r}(w)}{\partial w} + \frac{\partial\pi}{\partial q_r}\frac{\partial\bar{q_r}(w)}{\partial w} + \frac{\partial\pi}{\partial p_s}\frac{\partial\bar{p_s}}{\partial w} + \frac{\partial\pi}{\partial q_s}\frac{\partial\bar{q_s}(w)}{\partial w} + \frac{\partial\pi}{\partial w} = 0$$

$$\frac{d\pi_w}{dw} = \frac{\partial\pi}{\partial p_r}\frac{\partial\bar{p_r}(w)}{\partial w} + \frac{\partial\pi}{\partial w} = 0$$

Because $\frac{\partial\pi}{\partial q_r} = 0$ and $\frac{\partial\pi}{\partial q_s} = 0$ due to the F.O.C in the optimization of $\bar{q_r}$ and $\bar{q_s}$. Also, $\frac{\partial\bar{p_s}}{\partial w} = 0$ because $\bar{p_s}$ is independent of w.

$$\frac{\partial\pi}{\partial p_r} = q_r$$

$$\frac{\partial\bar{p_r}(w)}{\partial w} = \frac{\alpha_2}{2}$$

$$\frac{\partial\pi}{\partial w} = -2cw - \tau_1 - \tau_2$$

$$\frac{d\pi}{dw} = \frac{\alpha_2[\beta_2(\alpha_1 + \alpha_2 w - c - \tau_2) - \gamma(1 + \tau_1 - \bar{\lambda})]}{4(\beta_1\beta_2 - \gamma^2)} - 2cw - \tau_1 - \tau_2$$

First Order Condition

$$\frac{d\pi}{dw} = 0$$

$$\frac{\alpha_2[\beta_2(\alpha_1 + \alpha_2 w - c - \tau_2) - \gamma(1 + \tau_1 - \bar{\lambda})]}{4(\beta_1\beta_2 - \gamma^2)} - 2cw - \tau_1 - \tau_2 = 0$$

$$\frac{\alpha_2^2\beta_2 w}{4PD_P} + \frac{\alpha_2[\beta_2(\alpha_1 - c - \tau_2) - \gamma(1 + \tau_1 - \bar{\lambda})]}{4PD_P} - 2cw - \tau_1 - \tau_2 = 0$$

Where

$$PD_P = \beta_1\beta_2 - \gamma^2$$

$$RM_P = \alpha_1 - c - \tau_2$$

$$w\left[2c - \frac{\alpha_2^2\beta_2}{4PD_P}\right] = \frac{\alpha_2[\beta_2 RM_P - \gamma(1 + \tau_1 - \bar{\lambda})]}{4PD_P} - \tau_1 - \tau_2$$

$$\bar{w} = \frac{\alpha_2[\beta_2 RM_P - \gamma(1+\tau_1-\bar{\lambda})] - 4(\beta_1\beta_2-\gamma^2)(\tau_1+\tau_2)}{8c(\beta_1\beta_2-\gamma^2)-\alpha_2^2\beta_2}$$

Karush-Kuhn-Tucker Conditions

Complementary slackness condition is not sufficient: $q_s > 0$ and $\lambda = 0$

$$\bar{w} = \frac{\alpha_2[\beta_2 RM_P - \gamma(1+\tau_1)] - 4PD_P(\tau_1+\tau_2)}{8cPD_P-\alpha_2^2\beta_2} \tag{A.1}$$

$$\bar{q_r} = \frac{2[2c[\beta_2 RM_P - \gamma(1+\tau_1)] - \alpha_2\beta_2(\tau_1+\tau_2)]}{8cPD_P-\alpha_2^2\beta_2} \tag{A.2}$$

$$\bar{q_s} = \frac{\beta_1(1+\tau_1)-\gamma RM_P}{2PD_P} - \frac{\alpha_2\gamma}{8cPD_P-\alpha_2^2\beta_2}\left[\frac{\alpha_2[\beta_2 RM_P-\gamma(1+\tau_1)]}{2PD_P} - 2(\tau_1+\tau_2)\right] \tag{A.3}$$

$$\bar{p_r} = \frac{(RM_P)(16cPD_P-\alpha_2^2\beta_2) - 2\alpha_2[\alpha_2\gamma(1+\tau_1)+4PD_P(\tau_1+\tau_2)]}{4(8cPD_P-\alpha_2^2\beta_2)} \tag{A.4}$$

$$\bar{p_s} = \frac{1-\tau_1}{2} \tag{A.5}$$

Second Order Condition

S.O.C. to verify under which condition w is a maximum in the model.

$\frac{d\pi}{dw} = 0$

Second derivative The second derivative shows the condition for which $\bar{w}$ is a maximum. Otherwise, the optimum shows a minimum point instead. Indeed, the optimal point is a maximum if the second derivative is negative.

$$\frac{d\pi}{dw} = \frac{\alpha_2[\beta_2(\alpha_1+\alpha_2 w - c - \tau_2) - \gamma(1+\tau_1-\lambda)]}{4(\beta_1\beta_2-\gamma^2)} - 2cw - \tau_1 - \tau_2$$

$$\frac{d^2\pi}{dw^2} = \frac{\beta_2\alpha_2^2}{4(\beta_1\beta_2-\gamma^2)} - 2c$$

So, for the second derivative to be negative, $\frac{\beta_2\alpha_2^2}{4(\beta_1\beta_2-\gamma^2)} < 2c$. This indicates the following assumptions:

$$c > \frac{\beta_2\alpha_2^2}{8(\beta_1\beta_2-\gamma^2)}$$

$$(\beta_1\beta_2-\gamma^2) > \frac{\beta_2\alpha_2^2}{8c}$$

$$PD_P > \frac{\beta_2\alpha_2^2}{8c}$$

$\frac{d\pi}{dq_r} = 0$

$$\frac{d\pi}{dq_r} = \alpha_1 + \alpha_2 w - 2\beta_1 q_r - 2\gamma q_s - c - \tau_2$$

$$\frac{d^2\pi}{dq_r^2} = -2\beta_1$$

$\bar{q_r}$ is always a maximum.

$\frac{d\pi}{dq_s} = 0$

$$\frac{d\pi}{dq_s} = 1 + \tau_1 - 2\beta_2 q_s - 2\gamma q_r$$

$$\frac{d^2\pi}{dq_s^2} = -2\beta_2$$

$\bar{q_s}$ is always a maximum.

APPENDIX B SOCIAL OPTIMUM

Social Welfare Function

$$SW = CS + Profits + Tax\ Revenues - Externalities$$

$$SW = CS_{q_r} + CS_{q_s} + \pi + \tau_1(q_s - w) + \tau_2(q_r + w) - D(q_r + w)$$

$$\begin{aligned} SW_{(w,q_r,q_s)} = & \int_0^{q_r} [p_r(q_r) - \tilde{p_r}]dq_r + \int_0^{q_s} [p_s(q_s) - \tilde{p_s}]dq_s \\ & + p_r q_r + p_s q_s - c(q_r + w^2) \\ & - D(q_r + w) \end{aligned}$$

Where D is a constant for the environmental damage caused by the externalities.

Inverse demand function

Regular price

$$p_r(q_r) = \alpha_1 + \alpha_2 w - \beta_1 q_r - \gamma q_s$$

$$p_r(0) = \alpha_1 + \alpha_2 w$$

$$p_r^* = \alpha_1 + \alpha_2 w^* - \beta_1 q_r^* - \gamma q_s^*$$

Price on sale

$$p_s(q_s) = 1 - \gamma q_r - \beta_2 q_s$$

$$p_s(0) = 1$$

$$p_s^* = 1 - \gamma q_r^* - \beta_2 q_s^*$$

CS_{q_r} **triangle area**

$$\begin{aligned} CS_{q_r} &= \int_0^{q_r} [p_r(q_r) - \tilde{p_r}] dq_r \\ &= \frac{[p_r(0) - \tilde{p_r}] q_r}{2} \\ &= \frac{[(\alpha_1 + \alpha_2 w - \gamma q_s) - (\alpha_1 + \alpha_2 w - \beta_1 q_r - \gamma q_s)] q_r}{2} \\ &= \frac{\beta_1 q_r^2}{2} \end{aligned}$$

CS_{q_r} **triangle area**

$$\begin{aligned} CS_{q_s} &= \int_0^{q_s} [p_s(q_s) - \tilde{p_s}] dq_s \\ &= \frac{[p_s(0) - \tilde{p_s}] q_s}{2} \\ &= \frac{[1 - \gamma q_r - (1 - \gamma q_r - \beta_2 q_s)] q_s}{2} \\ &= \frac{\beta_2 q_s^2}{2} \end{aligned}$$

Constraint

$$q_s \leq w$$

Lagrangian

$$max_{(w,q_r,q_s)}\ SW$$

$$\text{subject to } q_s \leq w$$

$$\mathcal{L}_{(w,q_r,q_s,\lambda)} = SW_{(w,q_r,q_s)} + \lambda(w - q_s)$$

$$\begin{aligned} L_{(w,q_r,q_s,\lambda)} = \int_0^{q_r} [p_r(q_r) - p_r^*]\, dq_r + \int_0^{q_s} [p_s(q_s) - p_s^*]\, dq_s \\ p_r q_r + p_s q_s - c(q_r + w^2) \\ - D(q_r + w) + \lambda(w - q_s) \end{aligned}$$

$$\mathcal{L}_{(w,q_r,q_s,\lambda)} = \frac{\beta_1 q_r^2}{2} + \frac{\beta_2 q_s^2}{2} + p_r q_r + p_s q_s - c(q_r + w^2) - D(q_r + w) + \lambda(w - q_s)$$

First Order Condition

Quantity related to trends (w)

$$\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{dw} = \alpha_2 q_r + \lambda + 2cw - D = 0$$

$$w^* = \frac{\alpha_2 q_r - \tau_1 - \tau_2 - D + \lambda^*}{2c} \tag{B.1}$$

Quantity sold at regular price (q_r)

$$\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{dq_r} = \alpha_1 + \alpha_2 w - \beta_1 q_r - 2\gamma q_s - c - D = 0$$

Quantity sold on sale (q_s)

$$\frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{dq_s} = 1 - 2\gamma q_r - \beta_2 q_s - \lambda = 0$$

Lagrangean multiplier (λ)

$$\lambda \frac{d\mathcal{L}_{w,q_r,q_s,\lambda}}{d\lambda} = \lambda(w - q_s) = 0, \lambda \geq 0$$

Saturated KKT conditions ($\lambda > 0$ and $q_s = w$)

$$w = q_s$$

$$max_{(w,q_r)}\ SW$$

$$SW_{(w,q_r)} = CS_{q_r} + CS_{q_s} + p_r q_r + p_s + q_s - c(q_r + w^2) - D(q_r + w)$$

$$SW_{(w,q_r)} = \frac{\beta_1 q_r^2}{2} + \frac{\beta_2 w^2}{2} + q_r(\alpha_1 + \alpha_2 - \beta_1 q_r - \gamma w) + w(1 - \gamma q_r - \beta_2) - c(q_r + w^2) - D(q_r + w)$$

Social Optimum

Where,

$$PD_{FB} = \beta_1\beta_2 - 4\gamma^2$$

$$RM_{FB} = \alpha_1 - c - D$$

$$\lambda^* = \frac{2c(1+\tau_1+2q_r^*) - \beta_2(\alpha_2 q_r^* - \tau_1 - \tau_2 - D)}{2c+\beta_2} \tag{B.2}$$

$$w^* = \frac{\alpha_2[\beta_2(\alpha_1 - c - \tau_2 - D) - 2\gamma(1+\tau_1-\lambda^*)] - PD_{FB}(\tau_1+\tau_2+D-\lambda^*)}{2cPD_{FB} - \alpha_2^2\beta_2} \tag{B.3}$$

$$q_r^* = \frac{\beta_2(\alpha_1 + \alpha_2 w^* - c - \tau_2 - D) - 2\gamma(1+\tau_1-\lambda^*)}{PD_{FB}} \tag{B.4}$$

$$q_s^* = \frac{\beta_1(1+\tau_1-\lambda^*) - 2\gamma(\alpha_1 + \alpha_2 w^* - c - \tau_2 - D)}{PD_{FB}} \tag{B.5}$$

When the KKT conditions are inactive ($\lambda = 0$ and $q_s > 0$), the expressions of quantities and prices at the social optimum become:

$$w^* = \frac{\alpha_2[\beta_2 RM_{FB} - 2\gamma] - PD_{FB} \times D}{2cPD_{FB} - \alpha_2^2\beta_2} \tag{B.6}$$

$$q_r^* = \frac{2c[\beta_2 RM_{FB} - 2\gamma] - \alpha_2\beta_2 D}{2cPD_{FB} - \alpha_2^2\beta_2} \tag{B.7}$$

$$q_s^* = \frac{\beta_1 - 2\gamma RM_{FB}}{PD_{FB}} - \frac{2\alpha_2\gamma}{2cPD_{FB} - \alpha_2^2\beta_2}\left[\frac{[\beta_2 RM_{FB} - 2\gamma]}{PD_{FB}} - D\right] \tag{B.8}$$

$$\begin{aligned} p_r^* =& \frac{\beta_1\gamma + (\beta_1\beta_2 - 2\gamma^2)(c+D)}{PD_{FB}} - \frac{2\gamma^2}{PD_{FB}}\left[\alpha_1 + \frac{\alpha_2^2[\beta_2 RM_{FB} - 2\gamma]}{2cPD_{FB} - \alpha_2^2\beta_2}\right] \\ &+ \frac{2\alpha_2\gamma^2 D}{2c_1 PD_{FB} - \alpha_2^2\beta_2} \end{aligned} \tag{B.9}$$

$$\begin{aligned} p_s^* =& 1 + \frac{\beta_2\gamma}{2cPD_{FB} - \alpha_2^2\beta_2}\left[2cRM_{FB} - \frac{\alpha_2}{2c - \alpha_2^2\beta_2}\left(\frac{2\alpha_2\gamma}{PD_{FB}} + D\right)\right] \\ &- \frac{(\beta_1\beta_2 - 2\gamma^2)}{PD_{FB}} \end{aligned} \tag{B.10}$$

When the KKT conditions are active ($\lambda > 0$ and $q_s = w$), the expressions of quantites and prices at the social optimum become:

$$w^* = \frac{\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2} = q_s^* \tag{B.11}$$

$$q_r^* = \frac{\alpha_1 - c - D}{\beta_1} + \frac{(\alpha_2 - 2\gamma)}{\beta_1}\left[\frac{\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2}\right] \tag{B.12}$$

$$p_r^* = \frac{\gamma[\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)]}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2} + c + D \tag{B.13}$$

$$p_s^* = 1 - \frac{\gamma(\alpha_1 - c - D)}{\beta_1} - [\gamma(\alpha_2 - 2\gamma)]\left[\frac{\beta_1(1-D) + (\alpha_2 - 2\gamma)(\alpha_1 - c - D)}{\beta_1(\beta_2 + 2c) - (\alpha_2 - 2\gamma)^2}\right] \tag{B.14}$$

Second Order Condition

$$SW_{(w,q_r,q_s)} = \int_0^{q_r} [p_r(q_r) - \bar{p_r}]dq_r + \int_0^{q_s} [p_s(q_s) - \bar{p_s}]dq_s + \pi - D(q_r + w) \tag{B.15}$$

$$SW_{(w,q_r,q_s)} = \frac{q_r[\beta_1 q_r + \gamma q_s]}{2} + \frac{q_s[\gamma q_r + \beta_2 q_s]}{2} + p_r q_r + p_s q_s - c(q_r + w^2) - \tau_1(w - q_s) - \tau_2(q_r + w) - D(q_r + w) \tag{B.16}$$

Quantity related to trends (w)

$$\frac{dSW}{dw} = \frac{\alpha_2\gamma(\beta_2 q_s^* - 2\gamma q_r^*)}{PD_{FB}} - 2cw - \tau_1 - \tau_2 - D$$

$$\frac{d^2SW}{dw^2} = \frac{\alpha_2\gamma}{PD_{FB}}\left[\beta_2\frac{dq_s^*}{dw} - 2\gamma\frac{dq_r^*}{dw}\right] - 2c\frac{dw}{dw} - 0 - 0 - 0$$

$$\frac{d^2SW}{dw^2} = -\frac{4\alpha_2^2\beta_2\gamma^2}{PD_{FB}^2} - 2c$$

Since $\frac{d^2SW}{dw^2} < 0$, then w^* is always a maximum.

Quantity sold at regular price (q_r)

$$\frac{dSW}{dq_r} = [p_r(q_r) - p_r^*] + 0 - \frac{d\pi}{dq_r} - \frac{d[D(q_r + w)]}{dq_r}$$

$$\frac{dSW}{dq_r} = \alpha_1 + \alpha_2 - \beta_1 q_r - 2\gamma q_s - c - \tau_2 - D$$

$$\frac{d^2SW}{dq_r^2} = -\beta_1$$

Here, $\frac{d^2SW}{dq_r^2} < 0$, so q_r^* is always a maximum.

Quantity sold on sale (q_s)

$$\frac{dSW}{dq_s} = 0 + [p_s(0) - p_s^*] + \frac{d\pi}{dq_s} - 0$$

$$\frac{dSW}{dq_s} = 1 + \tau_1 - 2\gamma q_r - \beta_2 q_s$$

$$\frac{d^2SW}{dq_s^2} = -\beta_2$$

Here, $\frac{d^2SW}{dq_s^2} < 0$, so q_s^* is always a maximum.