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POLYTECHNIQUE MONTRÉAL
affiliée à l'Université de Montréal

Workload Equity in Vehicle Routing Problems Over Multiple Periods

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Mémoire présenté en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*
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Ce mémoire intitulé :

Workload Equity in Vehicle Routing Problems Over Multiple Periods

présenté par **Najmeh NEKOOGHADIRLI**
en vue de l'obtention du diplôme de *Maîtrise ès sciences appliquées*
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DEDICATION

To my husband, Morteza;
for his endless love
To my parents, Manije and Habib;
for their endless support . . .

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RÉSUMÉ

Dans les problèmes de tournées de véhicules, la minimisation des coûts est rarement le seul objectif à considérer. D'autres objectifs doivent également être pris en compte dans la pratique, tel le partage équitable de la charge de travail entre les chauffeurs (équité, balancement des routes). On sait que la minimisation des coûts est susceptible de mener à des solutions où la charge de travail n'est pas distribuée équitablement. Autrement dit, des coûts additonnels doivent être encourus pour obtenir des solutions équitables. Il arrive souvent en pratique que l'équité doive être atteinte sur un certain nombre de périodes (e.g., journées) et non à chaque période. Dans ce mémoire, un problème de tournées de véhicules multi-périodes avec un objectif visant l'équité sur l'ensemble des périodes est résolu à l'aide d'une méthode en deux phases. Dans la première phase, une solution de distance minimale est produite pour le problème de tournées de véhicules associé à chaque période. Les routes ainsi obtenues sont ensuite distribuées entre les chauffeurs dans la seconde phase afin d'atteindre l'équité au niveau de la distance totale parcourue par chacun des chauffeurs sur l'ensemble des périodes. Une étude expérimentale démontre les bénéfices de cette approche sur des instances tests dérivées d'instances classiques pour le problème de tournées de véhicules avec contraintes de capacité. Les résultats démontrent en particulier que l'équité entre les chauffeurs peut être atteinte sans coût additionnel si le nombre de périodes est suffisamment grand.

ABSTRACT

In vehicle routing problems, cost minimization is rarely the only concern. There are other objectives that must be taken into account. One of these objectives is a fair distribution of the workload among drivers (equity, balance). It is known that minimizing operations costs is prone to lead to poorly balanced solutions, which means that better-balanced solutions lead to additional operations costs. In many real life problems, equity must be achieved over a certain number of periods (e.g., days), not within each period. In this thesis, a multi-period vehicle routing problem with an equity objective is addressed with a two-phase problem-solving methodology. In the first phase, a minimum-distance solution is produced for each period. The routes obtained are then combined in a second phase to achieve equity among drivers with regard to their total distance traveled over all periods. A computational study shows the benefits of this two-phase algorithm, based on instances derived from standard benchmark instances for the capacitated vehicle routing problem. The results show in particular that workload equity can be attained at no additional operations cost when the number of periods is sufficiently large.

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LIST OF SYMBOLS AND ACRONYMS

BCP	Branch-and-cut-and-price
CVRP	Capacitated Vehicle Routing Problem
HGS	hybrid genetic search
MAD	Mean Absolute Deviation
MILP	Mixed Integer Linear Programming
MVRPB	Multi-period Vehicle Routing Problem with workload Balance
PD	Pigou-Dalton Transfer Principle
PVRP	Periodic VRP
RCSPs	Resource-Constrained Shortest Paths
SD	Standard Deviation
TSP	Traveling Salesman Problem
Var	Variance
VRP	Vehicle Routing Problem

CHAPTER 1 INTRODUCTION

These days, numerous businesses operate in the transportation, distribution, and supply chain services. To stay alive in such a competitive environment, businesses need to focus on improving their distribution strategies. To this end, they must consider not only profit but also employees' and customers' satisfaction. Improving fairness among employees enhances their motivation to provide high-quality services to customers. Enhanced service quality to customers, in turn, generates more profit by favoring the emergence of loyal customers who may also recommend your services and products to other people. Thus, customer and driver satisfaction should not be overlooked.

1.1 Basic concepts

The classical VRP with capacity constraints is known in previous studies as the Capacitated Vehicle Routing Problem (CVRP). It is a variant of the Vehicle Routing Problem (VRP) which has been studied for many years by many researchers (Toth and Vigo, 2002). This problem involves a set of customers, each with a specific amount of demand, and a fleet of capacitated vehicles located at a central depot. In this problem, the demand of every customer must be served exactly once by a vehicle. The goal is to create a set of feasible (i.e., capacity obedient) routes, one for each vehicle, that start and end at the depot and that minimize the cost (i.e., total distance traveled by the vehicles). Since the vehicles have a fixed capacity, multiple routes are typically needed to serve all customers. This combinatorial optimization problem is an extension of the Traveling Salesman Problem (TSP) where a single uncapacitated vehicle travels a single route to serve all customers at minimum cost. These problems are NP-hard which means that no polynomial-time algorithm is known to solve them. Consequently, solving any VRP variant requires heuristic approaches when large-sized instances are considered (Ribeiro and Lourenço, 2001; Vidal et al., 2013).

There is an extensive and diverse variety of VRPs reported in the literature (Chatonnay et al., 2014). They can be divided into different categories depending on their characteristics such as the underlying network, the demand of customers, the fleet of vehicles, the objective, the constraints, etc.

Multi-objective VRPs have also been considered in a number of studies. Multi-objective VRPs are variants of the VRP where different, often conflicting, objectives are considered. These problems optimize the cost as well as other objectives related to vehicles, drivers, or customers. For example, customer satisfaction and timely delivery are as important or even more important than operational costs in some sectors like the delivery of perishable goods. In other sectors, fair workload distribution among drivers is a crucial issue. The main reason for studying multi-objective VRPs is to better represent real-life problems (Jozefowiez et al., 2008a). Solving multi-objective problems lead to a set of trade-off solutions called non-dominated or Pareto-optimal solutions.

Many different objectives have been considered in the literature on VRP and its variants. Generally speaking, there are two large classes of objectives: monetary (cost) and non-monetary (service quality, reliability, consistency, ...). Monetary objectives are related to money (revenue, cost) or any equivalent of it, like distance or travel time. Non-monetary objectives, although not directly related to money, can be decisive for the medium or long-term viability of an organization, especially if they can be achieved at little additional cost (Matl et al., 2018). In recent work, Vidal et al. (2020) divide the objectives used in VRPs into seven groups:

- *Profitability:* alternative cost structures, profits, outsourcing;
- *Service quality:* cumulative objectives, on time service, service fulfillment;
- *Equity:* workload balance, service equity, collaborative planning;
- *Consistency:* customer-oriented, temporal, delivery;
- *Simplicity:* route compactness, geographical separation of routes, navigation complexity;
- *Reliability:* expected losses, probability of failure;
- *Externalities:* nocive emissions, safety risks, noise and congestion.

It should be noted that other ways to categorize objectives are reported in some older papers like Jozefowiez et al. (2008b). Without going into details, the latter divides the objectives among those that are related to routes, node/arc activities and resources.

- *Objectives related to routes:* cost, profit, makespan, route balancing, utilization of available capacity, risk aversion;

- *Objectives related to node/arc activities:* number of violated constraints, customer and/or driver's waiting time due to earliness or lateness, customer satisfaction, coverage, customer-driver consistency;

- *Objectives related to resources (vehicles and goods):* number of vehicles (since less vehicles leads to less investment costs, less emissions of CO_2 and less drivers' salaries), risk aversion, utility or disutility measures.

Multi-period VRPs represent another class of VRPs, which are defined over a planning horizon of several periods. A frequency of service (i.e., a subset of time periods) is first defined for each customer over the planning horizon, as well as a corresponding demand. The objective is to minimize the total cost over the planning horizon. An important point is that decisions made in one period are not independent of decisions made in other periods (Coene et al., 2010). Multi-period VRPs are surveyed and classified in Mourgaya and Vanderbeck (2006). There are also multi-objective VRPs that are reported in the literature for problems defined over a planning horizon that spans multiple time periods (Bansal and Goel, 2018).

1.2 Equity and VRP

Many companies in various routing-related industries seek to provide high-quality service and a balanced workload to improve customer or driver satisfaction. Equity is one of those objectives. Equity tries to provide fairness among different players in the distribution network. In many studies, fairly allocating workload between drivers is called workload equity. Fairly distributing services or products among customers or, when there is more than one shipper, fairly sharing other resources among shippers are other types of equity objectives that are introduced in the literature review.

1.2.1 Workload equity and VRP

This section focuses on workload equity in the VRP and its implementation in various problems. As mentioned in Section 1.2, the purpose of workload equity is the fair distribution of workload resources among drivers.

According to the terminology of Matl et al. (2018), an equity metric refers to the type of workload to be balanced among routes (e.g., served demand, distance traveled). An equity measure provides a value for a given allocation of workloads among drivers, like the difference between the highest and lowest workloads. An equity objective is then obtained by combining a metric with a measure. Bi-objective VRPs with cost as the first objective and equity as the second objective are called VRP with "route balancing" (Jozefowiez et al., 2007, 2009). It should be noted that some studies rather account for equity through constraints.

There are different types of equity measures introduced in previous studies (Matl et al., 2018; Lozano et al., 2016; Halvorsen-Weare and Savelsbergh, 2016), like Min-Max, where the route with the largest workload is minimized, Range which is the difference between the maximum and minimum workloads, Mean Absolute Deviation (MAD) which is the average of the absolute differences between each workload and the average workload, etc. Different equity measures, each with its pros and cons, are presented in detail in the literature review. The equity objective is important, not only because it allows a fair distribution of the workload among drivers, which leads to the driver and customer satisfaction, but also because it produces more robust solutions when unexpected events occur, like a sudden demand increase among customers, and reduces bottlenecks (Mourgaya and Vanderbeck, 2007).

1.3 Problem statement- multi-period VRP with workload balance

A cost increase is typically observed when workload equity is taken into account in classical single-period VRPs (Matl et al., 2019). Yet, it is likely that considering equity over multiple periods will alleviate this effect and the cost-related objectives will not grow as much because over multiple periods, we have much more flexibility for balancing the workload of drivers.

This thesis addresses a multi-period VRP, where distance-based workload equity must be achieved over the whole planning horizon. This is called the multi-period VRP with workload balance (MVRPB). Thus, even if it is not necessary for the workload to be equal in each period, we want to attain as much as possible workload equity among drivers over the planning horizon. This is a concern since there are many circumstances in real life where the workload is defined over a rather long planning horizon (just think of nurse scheduling problems, for example).

In the MVRPB, different subsets of customers need to be visited in different periods of the planning horizon, with possibly different demands. Thus, we have a different VRP associated with each time period of the planning horizon, where each VRP is defined over the subset of customers that have a demand for that period. The goal is to generate routes in each time period while accounting for the total distance traveled by all vehicles over the planning horizon, as well as a form of equity among the drivers with regard to the total distance traveled by each one of them. To be more precise, the equity objective minimizes the maximum vehicle workload (i.e., total distance traveled by a vehicle over the planning horizon) through the Min-Max measure. As previously mentioned, equity among drivers only applies to the whole planning horizon, not to individual time periods. Also, each driver is associated with at most one route in a given time period, while each route needs to be associated with a driver.

1.4 A brief overview of the proposed solution method

When workload equity is sought in single-period VRPs, the routing costs tend to grow. But, by considering workload equity over multiple periods, this cost increase may be mostly alleviated. This issue has motivated us to develop a two-stage optimization approach for our problem in which in the first step, a CVRP is solved in each time period while minimizing the cost, which is the total distance traveled by the vehicles. Then, in a second step, the routes obtained in the first step for each time period are assigned to the drivers to attain workload equity, while making sure that each driver is assigned to at most one route from each time period. Thus, efficiency is achieved in the first step and equity in the second step. We obtain the cost optimal routes in the first phase. And, then we provide a balance workload among them in the second phase. We can't guarantee that there are no other alternative cost optimal routes in the first phase. But we can guarantee that we provide an equal workload among the routes obtained in the first phase. We refer to these steps as (1) route optimization, and (2) multi-period workload balancing.

The computational results obtained with the two-step optimization approach show that equity can be reached over multiple periods without increasing the cost if the number of periods is sufficiently large.

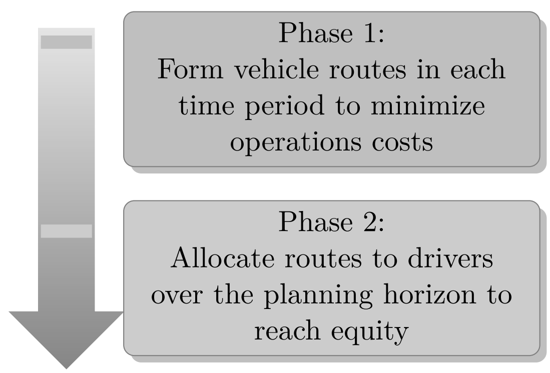

Figure 1.1 Two-phase approach

To solve the problem in the first and second steps, we take advantage of VRPSolver (Pessoa et al., 2020), which provides a generic branch-and-cut-and-price (BCP) framework that can solve many classes of MILPs. Our test instances come from a CVRP benchmark introduced in Uchoa et al. (2017). In the first step, since an individual CVRP must be solved in each period, VRPSolver provides a powerful exact solver for this purpose. VRPSolver is also used in the second step by modeling the multi-period workload balancing problem as a bin packing problem. Adapting VRPSolver to address bin packing problems, along with a binary search to appropriately determine the size of the bins, proves to be an efficient approach for the second step.

CHAPTER 2 LITERATURE REVIEW

For the first time, Dantzig and Ramser (1959) introduced the VRP in their paper and called it the truck dispatching problem. Since then, several papers presented various models and solution approaches for the VRP and its variants. Although minimizing the total length of the routes, as well as the number of vehicles used, are the most common objectives in previous studies (Tang and Miller-Hooks, 2006), other objective functions are considered to address the needs of real-life applications. In the following, we will take a look at these different objectives in the context of VRPs.

2.1 Objectives for VRPs

Monetary objectives can not be seen as pure costs only, where pure costs refer to routing costs that should be minimized, like total distance, total travel time, etc. Sometimes, routing costs need to be considered along with other measures. For example, in Baldacci et al. (2020), the authors are interested in minimizing the total cost per load and also in maximizing the total profit associated with each route. Stavropoulou et al. (2019) consider the most profitable customers to be the most frequent ones. Then, the objective is to determine vehicle routes that maximize the net profit (difference between total gain and total cost).

Due to high demand, a company may outsource its shipping and delivery activities to subcontractors leading to a special case of VRP. In this regard, Gahm et al. (2017) study a VRP in which the shipping company can use a private fleet, common carriers, or a combination of both to maximize profit. In this problem, the common carriers propose a volume discount and the subcontractor provides two vehicle rental options.

Monetary objectives are not always the main concern, such as in healthcare logistics, relief operations in natural disasters, humanitarian logistics, or public transportation. Furthermore, there are objectives that are looking neither for cost minimization nor driver and customer satisfaction nor providing service to the public; in practice, logistics and transportation activities have undesirable side effects, which are called externalities (e.g., nocive emissions). Thus, some objectives take into account the efforts to reduce the side effects of transportation and logistics activities.

Cumulative objectives are typically used in service-focused environments. For example, Expósito Márquez et al. (2019) examine different measures for service quality objectives based on time metrics. The cumulative objective function measures the total time that customers have to wait to be served from the starting time of the routes. The minimization of the sum of arrival times at each individual customer tends to favor a good service distribution over time. Huang et al. (2012) use a cumulative objective to minimize total arrival times or customers' waiting time. Energy minimizing VRP and school-bus routing problems are two other applications where cumulative objectives are used (Kara et al., 2008). Expósito Márquez et al. (2019) point out that on-time performance is aimed at minimizing the earliness or lateness at customer locations (Soman and Patil, 2020; Stavropoulou et al., 2019); it is particularly useful in passenger transportation, like school bus routing problems. Cumulative objectives are also found in other time-sensitive applications, like courier services. In practice, in service-focused objectives, it is not always possible to fulfill all requests in terms of delivery time or amount delivered to each customer. They play a role when a minimum level of service to customers is required. For example, Orlis et al. (2019) implement service level as a soft constraint and as a minimum allowable percentage of fulfilled requests.

Equity is aimed at a fair distribution of resources, responsibilities, and benefits among drivers or customers. Fair distribution of the workload resources among drivers is called workload equity. It maintains employee satisfaction, reduces overtime, and reduces bottlenecks in resource utilization. The workload resources can correspond to travel time (distance), the number of served customers, served demand, etc. (Matl et al., 2018). In Huang et al. (2012), one of the objectives is fair distribution of services or products among customers in humanitarian relief logistics. They focus on equitable service to all aid recipients, which is referred to as service equity. Tinoco et al. (2017) share the capacity of the fleet of vehicles (ships) between two shippers to fairly allocate the cost reduction among them, which is known as co-loading or collaborative planning; in their problem, the long-term goal is reducing the number of vehicles, consequently costs and emissions.

Consistency can be important for drivers and regular (commercial) customers (Kovacs et al., 2014). Arrival time consistency is used in particular to obtain predictable service start times for regular customers (e.g., similar service time every day, as in Campelo et al. 2019). Person-oriented consistency means the preference of regular customers to be served by familiar faces. Similarly, drivers like to serve the same customers along well-known routes in well-known service areas. Furthermore, when the drivers are familiar with their routes, learning requirements decrease and efficiency increases (Rodríguez-Martín et al., 2019). Delivery consistency is concerned with the delivery of a consistent quantity of goods or a consistent level of service (Coelho et al., 2012). On the other hand, inconsistency may be a desirable objective in the transportation of valuable objects to reduce the risk of robbery (Kovacs et al., 2014). Promoting the visual attractiveness of routes in VRP is another objective that simplifies practical fulfillment effectively. This can be implemented through compactness maximization, minimization of route overlap and crossing, and route complexity minimization (Rossit et al., 2019). There are several classifications for compactness but in a few words, a route with customers that are geographically clustered is more appealing (Rossit et al., 2019). For example, Lei et al. (2015) maximize compactness and minimize dissimilarity of districts in different periods.

Rossit et al. (2016) minimize cost with respect to an upper bound on the number of shared nodes between routes. When there is little or no overlap and crossing of routes, they are said to be separated. Separated routes make coordination easier because a local change in a route does not lead to a dramatic change in other routes. Also, distribution companies prefer routes that are easier to navigate. This objective can be evaluated, for example, by the number of right or left turns (Vidal, 2017).

In real-life problems, identifying reliable routes can be a concern due to the occurrence of unexpected events. A good way to account for uncertainty and find cost-effective routes is through stochastic or robust VRP models by representative scenarios, probability distributions, or uncertainty sets. Travel time, demand at customers, and the occurrence of new requests are the main sources of uncertainty in VRPs (Vidal et al., 2020). Salavati-Khoshghalb et al. (2019) minimize the expected cost to serve stochastic demand and Zhang et al. (2019a) maximize the joint probability of punctual arrivals and minimize the potential risk of failure.

Logistics and transportation in VRP have side effects. One of the disadvantages of road transportation is the increase in the amount of greenhouse gas emissions. Bektaş and Laporte (2011) study the Pollution-Routing Problem that minimizes not only common objectives of VRP such as time, distance, and cost but also the total greenhouse emissions. Among all factors, load and speed can vary while traversing an arc, so the authors report the effect of load and speed on gas emissions. They show that load and speed have an impact on the amount of energy consumed and gas emissions and that there is no direct relationship between cost and emission objectives. Wang et al. (2019) also minimize fuel consumption (based on fuel type, vehicle type, and road characteristics) and drivers' wage. Besides, when hazardous materials are transported, the most important concern is to minimize the risk or severity of an accident which is in close relationship with the design of a route (Vidal et al., 2020). Grabensch-weiger et al. (2018) minimize load-dependent and load-independent emissions and minimize disturbance of routes.

It is worth mentioning that non-monetary objectives such as service quality, reliability, and consistency will lead to better performance in terms of cost at a strategic or tactical level.

2.1.1 Overview of problems based on different objective functions

Table 2.1 provides a compact view of the different kinds of objectives and corresponding solution methods reported in the literature for VRPs.

Table 2.1: An overview of recent problems with different objective functions and assumptions

Class	Author	Application	Approach	Solution method	Objective function
Performance ratio	et al. (2020)	with fractional objective function	Single objective	Exact method	Min total cost per load / Max profit over time
Profit Maximizing	Stavropoulou et al. (2019)	VRP with consistency constraints	Single objective	Adaptive Tabu Search	Selection of most profitable customer (difference between gain and cost)
Outsourcing	Gahm et al. (2017)	VRP with private fleet and common carrier offering volume discounts, and rental options	Single objective	VNS	Minimize the sum of fixed vehicle costs, travel costs, and common carrier costs
Cumulative objectives	Huang et al. (2012)	Relief routing problem	Joint optimization	GRASP	Min cost, total arrival times (customers' waiting time) and inequity
On-time performance	Soman and Patil (2020)	Service Quality in VRP	single objective	A scatter search method	Minimize the sum of inventory holding, transportation, tardiness, and backorder costs
Service fulfillment	Orlis et al. (2019)	Service level fulfilment in coin and banknote distribution	Soft constraint	Branch-and-cut	Maximize the difference between collected revenues and transportation costs
Workload balance	Zhang et al. (2019b)	VRP with route balancing	Bi-objective	Multi-objective memetic algorithm	Min cost and inequality (longest tour and Range)
Service equity	Huang et al. (2012)	Relief routing problem	Joint optimization	GRASP	Min cost, total arrival times (customers' waiting time) and inequity

Table 2.1: An overview of recent problems with different objective functions and assumptions (continued and end)

Class	Author	Application	Approach	Solution method	Objective function
Collaborative planning	Tinoco et al. (2017)	Collaborative shipping by can-order policy to reduce the number of vehicles, costs and emissions	Single objective	Exact method	Fair allocation of cost reduction
Arrival time consistency	Campelo et al. (2019)	Servicing customers in a consistent TW minimize total cost	Constraint	Mathematical programming-based decomposition approach (Matheheuristic)	Serving customers inconsistent time while minimizing expected travel costs
Person oriented consistency	Rodríguez-Martín et al. (2019)	Drivers consistency in all periods of planning horizon	Constraint	Branch-and-cut	Minimize cost so that each customer is served by the same vehicle/driver each time
Delivery consistency	Coelho et al. (2012)	Consistency in quantity of delivered to each customer+ other features of consistency	Lower and upper bound constraints	ALNS metaheuristics	Minimize cost with all customers being served a specific amount
Compactness	Lei et al. (2015)	Minimizes compactness, dissimilarity, and equity measure	Weighted sum	Adaptive large neighborhood search	Minimize ratio of the territory's (route's) perimeter to the total perimeter of the service area, dissimilarity of districts in different periods and variance on average profit of the sales man
Separation	Rossit et al. (2016)	Increasing visual attractiveness to produce nicer routes which leads to real saving for companies	Constraint	Heuristic	Minimize cost with respect to upper bound on the number of shared nodes between routes

Table 2.1: An overview of recent problems with different objective functions and assumptions (continued)

Class	Author	Application	Approach	Solution method	Objective function
Separation	Constantino et al. (2015)	Assign tasks to vehicle routes to minimize overlap	Single objective/ Upper and lower bound constraint for overlapping	Heuristic	Minimize number of edges /nodes shared by two or more routes
Navigation complexity	Vidal (2017)	Minimizes number of turns in routes	Aggregation	Iterated local search (ILS)/ unified hybrid genetic search (UHGS) with extended neighborhoods	Min turn penalties and total cost
Expected cost or loss	Salavati-Khoshghalb et al. (2019)	Minimizes the expected cost to serve stochastic demand	Single objective	Integer L-shaped algorithm within a branch-and-cut algorithm	Minimize the expected cost
Risk of failure	Zhang et al. (2019a)	Maximizes the joint probability of punctual arrivals and minimizes the potential risk of failure	Single objective	Benders Decomposition	Maximize the joint probability of punctual arrivals / Min potential riskiness (Violation of deadlines)
Emissions	Wang et al. (2019)	Integrated production and vehicle routing problem	Multi-objective	Hybrid Tabu Search	Minimizes fuel consumption (based on fuel, vehicle type, and road characteristics) and drivers wage
Safety risk	Grabenschweiger et al. (2018)	Minimization of CO_2 emissions as well as disturbance to urban neighborhoods	Bi-objective	Three variants of e-constraint method as well as the balanced box method	Minimize load-dependent and load-independent emissions and minimize disturbance of routes

2.1.2 Interaction between objectives

In this section, we shortly review the conflicts and overlaps between some objective functions. Here, overlap means that the previous classifications are not that rigid and that some objectives can pertain to two or more of the categories mentioned by Vidal et al. (2020). Conflict refers to the effect of one objective on the solution structure of the other one.

There is some overlap between service equity and service quality objectives in the literature. For example, Huang et al. (2012) aim to provide quick, sufficient and equal service to increase service quality. Standard Deviation (SD) and Range measures are used in the objective to increase equity and effectiveness (quick and sufficient distribution). The objective of minimizing the length of the longest tour (makespan) leads to equity among drivers and customer satisfaction in dial-a-ride problems, that is, minimization of the makespan is fair from the last customer's perspective in terms of time (Jozefowiez et al., 2008b). Tang and Miller-Hooks (2006) examine the effect of objective functions in the context of humanitarian relief efforts, where the true goal is to minimize the arrival time of relief parcels. It is worth noting that minimization of total routing cost does not lead to the minimization of the latest arrival times or sum of arrival times. On the other hand, minimization of the latest arrival times or sum of arrival times has a small marginal cost and leads to equity. In some studies, there is an overlap between outsourcing and collaborative planning when there is an option of using outsourced vehicles to share the capacity between several manufacturers with the same customers but complementary food products (Sprenger and Mönch, 2012). Cumulative objectives can be advantageous both in service quality objectives and objectives related to energy consumption (Kara et al., 2008).

Huang et al. (2012) consider efficiency (cost minimization), effectiveness (quick and sufficient distribution), and equity objectives in a weighted sum objective, and show how they affect route structure and resource utilization. Their study shows that effectiveness and equity lead to similarities in route structure, as opposed to efficiency where the routes have a different structure. Bertazzi et al. (2015a) compare the optimal solution of Min-Max solutions and the classic Min-cost solutions from the worst-case perspective. Their results on a set of test problems confirm that in the worst case, the longest route of the optimal Min-cost solution is k times the longest route of the optimal Min-Max solution, and the Min-Max total distance is at most k times the Min-cost total distance, where k is the number of available vehicles.

These studies demonstrate that the choice of objective function plays a significant role not only in the application but also in the structure of the produced VRP solutions.

2.2 VRP and equal workload

This chapter reviews previous studies on the VRP, where equal workload among drivers is considered. This issue for the VRP emerged in the 1990s, almost 35 years after the VRP was introduced. To the best of our knowledge, more than 70 papers were published on this topic since the 1990s. In the following, we go through some of the single-period and most of the multi-period papers on this topic.

The first studies that considered workload equity evolved around single-period problems. The earliest work Bowerman et al. (1995) considered workload equity in a multi-objective single-period routing problem. They used a variance measure with route length and number of transported students metrics as equity objectives for an urban school routing problem. Since then, several papers studied workload equity among drivers in the VRP in a single period context. Golden et al. (1997) proposed the use of Min-Max measure for equity in their single-period CVRP, which was then solved by an adaptive memory heuristic. In their mode, Lee and Ueng (1999) sought to find the shortest travel path and the best possible balance in the vehicle working time simultaneously. The balance workload measure is a semi-variance measure, which minimizes the sum of the working time difference between each vehicle and the vehicle with the shortest working time in the single-period setting. Jozefowiez et al. (2002) used the Range measure to balance the length of the tours in a bi-objective single-period VRP model. They solved their model with a multi-objective heuristic algorithm. Zhang et al. (2019b) develop a multi-objective memetic algorithm to solve a single-period bi-objective VRP. The authors use a Min-Max measure along with a route cost metric. In Lehuédé et al. (2020), a single-period bi-objective VRP with cost minimization and lexicographic Min-Max measure using a route duration metric is solved with a multi-directional local search heuristic. Vega-Mejia et al. (2019) optimize equity among drivers in a multi-objective single-period vehicle loading problem with loading constraints in which weight-bearing strength of three-dimensional items is considered. For equity purposes, the objective function is a combination of the Range measure and served demand metric. Two other objectives are the minimization of the total time needed to deliver all the items and the minimization of the shift of the gravity center inside the container of each vehicle after unloading boxes at each stop along the delivery route.

After a while, and in order to solve problems with assumptions closer to real-world problems, workload equity was introduced over a longer multi-period planning horizon. Ribeiro and Lourenço (2001) is one of the first papers to consider equity over a scheduling horizon of many periods. The problem is solved using a weighted sum of three objectives consisting of cost, equity, and market shares. The equity objective minimizes the standard deviation of the drivers' workloads at the end of the planning horizon, where the workload corresponds to the total amount delivered by a driver. In this paper, only small-size instances were solved exactly with Lingo.

Blakeley et al. (2003) use the Range measure in a multi-period setting and develop a heuristic to solve an elevator maintenance company's problem, which assigns technicians to customers and schedules their routes. The authors minimize travel time, overtime, and unbalanced workload (travel times) within each period through a weighted sum approach.

Mourgaya and Vanderbeck (2007) examined a Periodic VRP (PVRP) from a tactical point of view using a hierarchical approach. In this problem, the dates of customers' visits are determined to attain some service level. Then, customers are assigned to vehicles to achieve equity concerning deliveries to customers. Equity is reached through a constraint that bounds the served demand of each cluster of customers (less than vehicle capacity) in each time period. The problem is solved with a column generation-based heuristic. Groër et al. (2009) studied a multi-period VRP where a utility company must balance the daily workload of their meter readers. They set a lower and an upper bound on the number of customers and the length of each daily route. The solution method is a hybrid of heuristics and integer programming.

Jozefowiez et al. (2009) and Oyola and Løkketangen (2014) present different algorithms to solve a bi-objective single-period VRP in which they minimize the total length of routes as well as the difference between the maximal and the minimal route length (Range). In Gulczynski et al. (2011) a PVRP is addressed where an equal workload is looked for within each period. The equity metric is the number of served customers and the equity measure is the Range. The problem is then solved with an integer programming-based heuristic. Liu et al. (2013) formulated a PVRP with an equity objective for home health care logistics where three types of patients' demands should be satisfied. The equity objective is the Min-Max of travel time. The solution method is a combination of tabu search and local search.

Schönberger (2016) balance the route duration in each period of a multi-period VRP with pick-up requests. In this case, however, this is handled by introducing an upper bound constraint. Messaoudi et al. (2019) propose a PVRP in which multiple frequencies of visits are considered simultaneously for each customer while taking into consideration cost minimization, minimization of the number of stops to the same customer (consistency), and balanced workload objectives within each period. The equity objective of this study is the Min-Max of the service time. The authors report a case study in a hygiene service company with more than 6000 customers and 69,951 requests for visits over three months.

In Liu et al. (2020), the authors balance the workload among routes with the Min-max measure and route time metric on each day of the planning horizon for a periodic home health care assignment problem. A method based on the partition of the service area into regions is used to solve the problem. The results show the effectiveness of the algorithm to solve this problem.

Introducing equity considerations in the VRP was frequently done through a multi-objective approach, where the equity objective is typically in conflict with the distance minimization objective. Although equity is often applied through a multi-objective approach in the VRP, there are also studies in which equity is modeled with constraints (e.g., Fallah et al. (2020)) or as the primary objective (e.g., Lopez et al. (2014); Liu et al. (2013); Golden et al. (1997)).

It is worth pointing out that Blakeley et al. (2003); Mourgaya and Vanderbeck (2007); Groër et al. (2009); Gulczynski et al. (2011); Liu et al. (2013); Schönberger (2016); Linfati et al. (2018); Messaoudi et al. (2019) and Liu et al. (2020) implement equity for every single time period regardless of the drivers' workload over the total planning horizon.

As we can see, there is not so much work on multi-period VRPs that account for equity, even less when equity is considered over multiple periods. Even though the aforementioned papers consider multiple periods for workload balanced VRPs, most of them focus on case studies for very specific problems or solve only instances of small size. Due to the scarce number of studies in the literature on VRPs with workload equity over multiple periods (instead of equity within a single period), there is definitely room for research in this area, especially if we consider that the multi-period problems occur quite often in real life.

2.2.1 Desirable properties of inequality measures

In Matl et al. (2018), eight desirable properties of so-called inequality measures are proposed, where an inequality measure $I(\mathbf{x})$ represents the non-equity or unfairness of a given workload allocation. Thus, $\mathbf{x}$ is preferred over $\mathbf{x}'$ if $I(\mathbf{x}) < I(\mathbf{x}')$. In the following, X is the set of all feasible allocations.

1. (Inequality Relevance) If $x_i = x_j$ for all i and j in X, then $I(\mathbf{x}) = 0$, otherwise $I(\mathbf{x}) > 0$;
2. (Transitivity) Let $I(\mathbf{x}) \geq I(\mathbf{x}')$ and $I(\mathbf{x}') \geq I(\mathbf{x}'')$, then $I(\mathbf{x}) \geq I(\mathbf{x}'')$;
3. (Scale Invariance) $I(\mathbf{x}) = I(\lambda \mathbf{x})$ for $\lambda \in \mathbb{R} \setminus \{0\}$. (i.e., the unit of measure has no impact);
4. (Translation Invariance) Let $\alpha \in \mathbb{R}$, and u be a unit vector of length n. Then $I(\mathbf{x}) = I(\mathbf{x} + \alpha u)$. (i.e., the value remains the same if a constant is added to every workload);

5. (Population Independence) The measures which satisfy this property are not affected by the number of possible outcomes in the workload distribution. In population dependant measures a direct comparison of distributions of different sizes is not possible. To overcome this we can replicate each population a certain number of times such that both resulting populations are of the same size (e.g., the lowest common multiple of the original sizes);

6. (Anonymity or Symmetry) Let $\mathbf{x}'$ be a permutation of the elements in $\mathbf{x}$, then $I(\mathbf{x}) = I(\mathbf{x}')$;

7. (Monotonicity) Let $\mathbf{x}'$ be such that $x'_i = x_i + \delta_i$ for at least one i in $\mathbf{x}$. If $\delta_i \geq 0$ for all such i and $\delta_i > 0$ for at least one i, then $I(\mathbf{x}') \geq I(\mathbf{x})$;

8. (Pigou-Dalton Transfer Principle (PD)) Let $\mathbf{x}'$ be formed as follows: $x'_i = x_i + \delta, x'_j = x_j - \delta$, $x'_k = x_k$, for all $k \notin \{i, j\}$. If $0 \leq \delta < x_j - x_i$, then $I(\mathbf{x}') \leq I(\mathbf{x})$. Thus a transfer of workload from j to i is beneficial. That is, a transfer from a shorter route to a longer route, other things remaining unchanged, leads to less equitable solutions (Karsu and Morton (2015)). A transfer is said to be progressive (favoring the worse-off party) if $0 \leq \delta < xj - xi$, and regressive otherwise.

With regard to the PD principle, it is important to distinguish two different types of metrics: those whose sum is constant over all solutions (e.g., total demand or number of customers in typical VRPs), and those whose sum is variable (e.g., total distance traveled):

Definition 1. (Constant/Variable Sum Equity metric). An equity metric is constant-sum if $\sum_{i=1}^{n} x_i$ is identical for all solutions $\mathbf{x} \in X$ and is variable-sum otherwise.

It should be noted that the PD principle applies in the case of constant-sum metrics. Another important concept when considering an equity objective is workload inconsistency:

Definition 2. (Workload Inconsistency). A solution $\mathbf{x}$ is workload inconsistent if there exists another solution $\mathbf{x}' \in X$ such that $\sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} x'_i$ and $x_i \geq x'_i$ for all $i = 1, \ldots, n$, but $I(\mathbf{x}) < I(\mathbf{x}')$.

This situation occurs in particular when a solution is preferred over another one with shorter routes (i.e., the length of one or more routes is increased to artificially achieve equity).

2.2.2 Common inequality measures

In the following, we present the inequality measures reported in previous studies. First, we look at six inequality measures examined by Matl et al. (2018).

- **Min-Max** $Max_{i=1}^{n}\{x_i\}$

  This measure is aimed at minimizing the maximum (or worst) workload. Clearly, this measure cannot discriminate between solutions with the same maximum workload. For example, if we assume four routes, the measure would not discriminate between workload allocations (20,15,10,5) and (20,10,10,10), while the second is clearly more equitable. Also, (19,19,11,1) would be preferred over (20,10,10,10). In both examples, the total workload over all routes is the same.
- **Lexicographic Min-Max**

  This measure not only minimizes the worst workload, but also the second-worst (subject to minimization of the first), the third-worst (subject to minimization of the first two), and so on. It addresses the issue of distinguishing between solutions with the identical maximum workload. Lehuédé et al. (2020) address a VRP with cost minimization and lexicographic Min-Max measure with a multi-directional local search heuristic.
- **Range** $Max_{i=1}^{n}\{x_i\} - Min_{i=1}^{n}\{x_i\}$

  Here, we minimize the Range which is the difference between the maximum and minimum workloads. It has some pros and cons. It is simple and easy to understand and apply. But it is not sensitive to the workload values between the minimum and maximum. Also, it does not account for the absolute workload values. For example (10,9,8,7) would be preferred over (5,4,2,1), even though all workloads are smaller in the latter.
- **Mean Absolute Deviation (MAD)** $1/n \sum_{i=1}^{n} \mid x_i - \bar{x} \mid$

  The MAD is the average of the absolute differences between each workload and the mean workload. As opposed to previous measures, MAD takes into account every workload, not just extreme values.
- **Standard Deviation (SD)** $\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$

  Despite the fact that this measure satisfies almost all axioms, the disadvantage is its computational complexity. Also, it is not as intuitive as the previous measures for decision-makers.
- **Gini Coefficient** $\frac{1}{2n^2\bar{x}} \sum_{i=1}^{n} \sum_{j=1}^{n} \mid x_i - x_j \mid$

  This coefficient has a value between 0 and 1. Once again, a major disadvantage is its computational complexity. Mandell (1991) has proposed an equivalent linear formulation of this measure.

Lozano et al. (2016) have also introduced some inequality measures that have not been mentioned previously. They are listed below.

- **All-min** $\sum_{i=1}^{n}(x_i - Min_{i=1}^{n}\{x_i\})$
  This measure minimizes the sum of the differences between a workload and the minimum workload over all workloads.
- **Variance (Var)** $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$
  This is basically the same as SD.
- **Rel** $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Max_{i=1}^{n}\{x_i\}-x_i}{Max_{i=1}^{n}\{x_i\}}\right)$
  This measure is aimed at minimizing the average relative deviation of the workload values from the maximum workload.
  Schwarze and Voss (2013) have also proposed other measures:

- **Rel'** $\sum_{i=1}^{n} \mid x_i - \bar{x} \mid$
  Here, we want to minimize the sum of absolute differences between the workload values and their average $\bar{x}$.
  In a bi-objective mixed capacitated general routing problem, Halvorsen-Weare and Savelsbergh (2016) introduce some other new measures, along with some measures already mentioned:
- $\sum_{i=1}^{n}(x_i - \bar{x})$
  This measure is similar to the variance measure. Its computational complexity is good and it takes into account all workloads.
- $\sum_{i=1}^{n} \mid x_i - x_{TG} \mid$
  For some real-life routing problems there may exist a target route distance that all routes should be close to. This is denoted as x_{TG} in the above formulation.
- **Max-Min** $Min_{i=1}^{n}\{x_i\}$
  In contrast with 3.2.1, this measure is aimed at maximizing the minimum workload. When balanced routes are not required and we are looking for solutions without too long or too short routes, the Min-Max and Max-Min measures can be used. These two measures can lead to completely different solutions. For instance, the solution obtained with Min-Max is close to the one obtained when minimizing the total distance traveled, while Max-Min is close to the Range measure.

2.3 Analysis and evaluation of equity objectives

In the literature, different inequality measures have been analyzed and compared. This is briefly surveyed in the following.

Baños et al. (2013) consider the balanced workload among vehicles (drivers) using a Range measure based on both traveled distance or served demand. Their results confirm the role of the metric used on the solutions produced.

Schwarze and Voss (2013) provide a comparison between six equity measures. Among them, Range, All-Min, and Rel involve a difference, that is, workload values are subtracted. Hence, minimization of these measures leads to maximization of workload values with a negative sign, thus increasing the probability of producing unwanted solutions. Besides, Rel' and Var are difficult to calculate because they are nonlinear. The measure Min-Max is linear and, furthermore, it does not include differences between workload values, which motivated the authors' choice for Min-Max.

Halvorsen-Weare and Savelsbergh (2016) examine the effect of different equity measures on the Pareto solutions in a bi-objective mixed capacitated general routing problem. Their conclusion is that the type of equity measure selected affects the size of Pareto optimal solutions as well as the solution produced.

Lozano et al. (2016) solve VRP benchmark instances with an evolutionary algorithm (EA) based on two mutation operators and seven inequality measures: All-Min, Range, Min-Max, Rel, Var, MAD, and Gini. Thus, 14 different configurations were compared. The experiments have shown that Var, MAD, and Gini performed better, even when compared with Range and Min-Max which are widely used.

Zhang et al. (2019b) claim that the Range measure leads to distorted solutions (i.e., non-TSP optimal at the route level or artificially balanced routes at the solution level). Accordingly, they use a Min-Max measure in their work.

Matl et al. (2018) conclude that the objective function by itself does not seem to have such a great impact on solution quality. But they argue about a possible correlation between the characteristics of a VRP instance and an appropriate equity objective. They claim that no measure satisfies every desirable property, and none is strictly better than the others over all relevant aspects (as shown in Table 2.2). Then, based on some previous theorems and observations, they conclude that:

- Monotonic measures $\Rightarrow$ Solutions which are not workload inconsistent
- Monotonic measures $\Rightarrow$ TSP optimal solutions

Table 2.2 Inequality measures and their properties

Property	Ineq. Rel.	Tran sitive	Scale Inv.	Translation Inv.	Population Ind.	Anon ymous	Monot onicity	P.D.
Min-Max		*			*	*	Weak	Weak
Lexi.Min-Max		*			*	*	Strong	Strong
Range	*	*		*	*	*		Weak
Mean abs. dev.	*	*		*	*	*		Weak
SD	*	*		*	*	*		Strong
Gini coeff.	*	*		*	*	*		Strong
All-min	*	*		*		*		Weak
Var	*	*		*	*	*		Strong
Rel	*	*	*		*	*		Weak
Rel'	*	*		*		*		Weak
$\sum_{i=1}^{n}(x_i - \bar{x})$	*	*		*		*		Weak
$\sum_{i=1}^{n} \mid x_i - x_{TG} \mid$		*				*		Weak
Max-Min	*	*		*	*	*		Weak

- TSP-optimal solutions $\Rightarrow$ Workload consistent or Workload inconsistent solutions
- Non-monotonic measures $\Rightarrow$ Workload consistent or Workload inconsistent solutions
- Non-monotonic measures $\Rightarrow$ TSP optimal or non-TSP optimal solutions

They confirm that cost-optimal VRP solutions are usually quite poorly balanced. On average, the longest route is about twice as long as the shortest one. On the other hand, they show that the marginal additional cost to improve equity is usually low. They also conclude that more sophisticated equity measures do not necessarily result in more reasonable trade-off solutions. In particular, monotonic measures, such as Min-Max or its lexicographic extension, are appropriate when workloads are based on variable-sum metrics (e.g., distance).

Matl et al. (2019) state that the structure of a solution mostly depends on the equity metric (workload), as opposed to the equity measure. They assert that for an equity objective with constant-sum resources all solutions are Pareto-optimal, without any consideration for the equity measure. Also, for an equity measure with a variable-sum workload, the Pareto-optimality of every allocation is not guaranteed, and choosing a monotonic measure plays a key role.

We summarize some of the results reported in their paper that are useful from our research point of view.

- Combining route distance (variable-sum) with non-monotonic measures can lead to an optimal solution with non-TSP-optimal routes or even an optimal solution where all routes are longer than those in a sub-optimal solution (workload inconsistency);

- The number of trade-off solutions increases with the complexity of an equity measure. Besides, balancing distance versus load, and balancing load versus the number of customers produces an increase in the number of trade-off solutions;
- For the same workload resource, most solutions found through an equity measure can be found with another equity measure. Also, for a given workload resource, solutions that optimize an equity measure tend to be of high quality for other equity measures as well.

CHAPTER 3 ORGANIZATION OF THE DOCUMENT

In this chapter, we present the thesis organization. Chapter 1 discusses the importance of driver satisfaction in vehicle routing problems. It also introduces the basic concepts of the vehicle routing problem (VRP) and its variants. Finally, workload equity, in particular multi-period workload equity, is discussed.

Chapter 2 introduces different objectives reported in the literature when solving VRPs. It reviews works on workload equity and introduces different measures and metrics proposed in the literature to reach equity. It shows in particular that few studies have addressed workload equity in a multi-period context, while none of them have examined its marginal cost. To fill this gap, we wrote the article to present a multi-period vehicle routing problem with an equity objective. This article has been submitted to *Computers and Operations Research* journal on 2022/06/29. Chapter 4, in Section 4.3, proposes the statement of this multi-period vehicle routing problem with an equity objective. A two-phase solution approach is then developed to solve this problem, as described in Section 4.4. A computational study in Section 4.5 shows the performance and benefits of this two-phase approach.

Chapter 5 provides a general discussion of our problem-solving methodology. Finally, Chapter 6 summarizes the contributions and limitations of this work. It also offers future research directions.

CHAPTER 4 ARTICLE 1: WORKLOAD EQUITY IN MULTI-PERIOD VEHICLE ROUTING PROBLEMS

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Abstract. An equitable distribution of workload is essential when deploying vehicle routing solutions in practice. For this reason, previous studies have formulated vehicle routing problems with workload-balance objectives or constraints, leading to trade-off solutions between routing costs and workload equity. These methods consider a single planning period; however, equity is often sought over several days in practice. In this work, we show that workload equity over multiple periods can be achieved without impact on transportation costs when the planning horizon is sufficiently large. To achieve this, we design a two-phase method to solve multi-period vehicle routing problems with workload balance. Firstly, our approach produces solutions with minimal distance for each period. Next, the resulting routes are allocated to drivers to obtain equitable workloads over the planning horizon. We conduct extensive numerical experiments to measure the performance of the proposed approach and the level of workload equity achieved for different planning-horizon lengths. For horizons of five days or more, we observe that near-optimal workload equity and optimal routing costs are jointly achievable.

Keywords: Vehicle routing problem, Multiple periods, Workload equity, Optimization

4.1 Introduction

Competition between companies in the transportation and distribution sectors has created a need to improve many business practices. Although minimizing distribution costs is essential, drivers' and clients' satisfaction should also not be neglected. Equitable workload assignments, especially, are fundamental to maintaining employee satisfaction at the workplace and can significantly impact the quality of services provided to clients. In recent works on vehicle routing problems with equity considerations, workload balancing is often treated as a second objective—along with cost minimization—in a bi-objective problem. In this context, Matl et al. (2019) reported that the marginal cost of balancing routes is reasonable in most cases: nearly 40% of Pareto-optimal solutions have an additional cost that does not exceed 10% of the minimum-cost solution. Solutions that account for equity also appear more robust against unexpected events, such as a sudden increase in demand, because bottlenecks are reduced (Matl et al., 2019; Mourgaya and Vanderbeck, 2007). Yet, such extra costs remain significant given that the transportation sector operates with very tight profit margins.

In most previous studies, we noted that the definition of workload equity was unnecessarily restrictive, as it focused on the balance between different drivers' workloads for each day separately. In practical situations, however, workload differences among drivers can be acceptable on each given day as long as the total workload over a longer time horizon (e.g., a week) remains equitable. To capture this notion, we formally define and study a multi-period VRP with workload balance (MVRPB). In this problem, customer requests are known on a longer planning horizon, and the goal is to find routes that optimize distance and workload balance over the entire planning horizon. To solve the MVRPB, we propose a two-phase solution approach. In the first phase, a solution with optimal distance is found for each period by optimally solving the corresponding VRPs. The resulting routes are then combined in a second phase to obtain equitable workloads among drivers based on the total distance traveled over the periods. We use this methodology to evaluate to which extent the length of the planning horizon can impact equitable solutions. Therefore, this work makes the following contributions:

1. We study workload balance among drivers over an extended planning horizon. With this viewpoint, we show that workload equity can be achieved with a very limited impact on economic efficiency.
2. We formally introduce the MVRPB and a simple and efficient two-phase approach for its solution. This approach also permits obtaining bounds on the best possible workload equity.

3. Through extensive numerical experiments, we demonstrate the performance of the proposed solution approach. Moreover, we measure (i) the gap between the equity level achieved by the two-phase method and perfect equity and (ii) the benefits of considering a longer planning horizon.

The remainder of this paper is organized as follows. Section 4.2 reviews related works for single and multi-period VRPs. Section 4.3 defines the MVRPB, and Section 4.4 describes the proposed two-phase solution approach. Section 4.5 presents a computational study based on test instances derived from well-known capacitated VRP benchmark instances. Finally, Section 4.6 concludes the paper and discusses research perspectives.

4.2 Related Studies

Single-period planning. The classical capacitated VRP (CVRP) seeks a set of routes starting and ending at a central depot and visiting a given set of clients (Vidal et al., 2020). Each client is characterized by a demand quantity and must be serviced in a single visit. The total demand of the clients over each route should not exceed the vehicle's capacity (considered to be identical for all vehicles). The objective of the problem is to minimize the total traveled distance. The CVRP is NP-hard as it generalizes the traveling salesman problem (TSP). Consequently, no optimal (i.e., exact) polynomial-time algorithm is known for its solution. Moreover, the best existing exact approaches can only solve medium-sized problems counting a few hundred customers in less than a few hours. Accordingly, heuristic approaches are widely used in practical settings (Ribeiro and Lourenço, 2001; Vidal et al., 2013).

Bowerman et al. (1995) were among the first to introduce an equity objective in the context of a school bus routing problem. They used a variance measure to balance the length of the drivers' routes and the number of students transported. In addition, they considered two additional criteria within a multi-objective framework. Since that work, numerous papers have considered workload balance objectives in single-period VRPs with different equity measures (see, e.g., Golden et al. 1997; Lee and Ueng 1999; Jozefowiez et al. 2009; Lopez et al. 2014; Oyola and Løkketangen 2014; Bertazzi et al. 2015b; Galindres Guancha et al. 2018; Va et al. 2018; Vega-Mejia et al. 2019; Zhang et al. 2019b; Lehuédé et al. 2020; Londono et al. 2021). The Min-Max measure and the difference between the maximum and minimum workloads (called Range by Matl et al. 2018) are most commonly used. When considering distance as the workload metric, Min-Max minimizes the longest route in a solution (see, e.g., Lopez et al. 2014), whereas Range minimizes the difference between the longest and shortest route (see, e.g., Londono et al. 2021). We refer the reader to Halvorsen-Weare and

Savelsbergh (2016), Lozano et al. (2016), and Matl et al. (2018) for a comprehensive list of equity measures.

No solution simultaneously optimizes cost and equity in most situations, especially when considering equity within a single period. Consequently, equity is often considered within multi-objective approaches, in which trade-off solutions have to be found between equity and other objectives such as distance. Many studies along this line refer to the resulting problem as the VRP with "route balancing" (Jozefowiez et al., 2007, 2009). Some studies have examined how equity objectives affect the shape of the Pareto front in a bi-objective context (Baños et al., 2013; Halvorsen-Weare and Savelsbergh, 2016; Lozano et al., 2016; Schwarze and Voss, 2013; Zhang et al., 2019b). Matl et al. (2018, 2019) extensively discussed six equity measures based on eight desirable properties. They claim that no measure satisfies every property or is strictly better than the others for all relevant properties. They also show that a monotonic equity measure such as Min-Max, which is based on a variable-sum metric such as distance, avoids non-TSP-optimal solutions and inconsistent solutions and therefore should be an objective of choice. A solution is non-TSP optimal if at least one route can be rearranged while improving its distance. Inconsistency occurs when a given solution is preferred over another solution even though all of its routes are longer. In both cases, the distance of one or more routes has been artificially increased to improve the equity objective.

Multi-period planning. Multi-period VRPs are defined over a time horizon of several periods and generally aim to minimize the total routing cost over all periods. In the periodic VRP (PVRP), each client is characterized by a visit frequency representing how many visits are requested over the planning horizon and a list of patterns representing acceptable visit-day combinations. Solving this problem requires selecting a visit pattern for each client and generating the routes for each period. As a consequence, the decisions made at each period become interdependent (Coene et al., 2010). Mourgaya and Vanderbeck (2006) presented several PVRP variants and classified them based on their objectives, constraints, and solution methods. Equity is one of the objectives discussed in that study.

Some studies considered multi-period VRPs and measured equity within each period. Papers in this category generally arise from real-life case studies and involve different workload equity objectives. Blakeley et al. (2003) studied the problem of an elevator-maintenance company that assigns technicians to clients. The objective was to minimize a weighted sum of travel time, overtime, and unbalanced workload in each period. To reach equity, the authors designed heuristics that optimize a Range measure on the travel times. Groër et al. (2009) studied a multi-period VRP to balance the workload of meter readers over a month for a utility company. To achieve a good balance, they set a lower and upper bound on the number of clients and the length of each daily route. They also constrain the deviation of a client's bill from one month to the next. Their three-stage methodology combines heuristics and integer programming. Gulczynski et al. (2011) presented a PVRP in which an equal workload is sought within each period. The overall objective is a weighted sum of distance and Range measure over the number of clients served. The problem was solved with an integer-programming-based heuristic.

Mourgaya and Vanderbeck (2006) designed a hierarchical heuristic to solve an industrial PVRP application with 16 658 visits over a time horizon of 20 days. At a tactical level, the method allocates each client to a combination of days and vehicles. The objective considered at this stage involves minimizing the maximal workload over the days and vehicles subject to additional geographical restrictions. At the operational level, the method minimizes the traveled distance.

Mourgaya and Vanderbeck (2007) then studied a tactical planning problem that required choosing visit days for clients subject to service level constraints. Once the visit days were selected, the clients were assigned to vehicles to achieve equity among drivers. At this stage, equity was modeled by a constraint that bounds the workload (served demand) associated with each cluster of clients. In another work, Linfati et al. (2018) developed a two-phase heuristic to balance the number of medication deliveries to patients among the drivers for each day of the planning horizon. They considered a Range measure within a weighted-sum objective that also accounts for extra hours, extra capacity, as well as daily and client clustering costs. Finally, Messaoudi et al. (2019) introduced a PVRP with different possible visit frequencies for each client. They relied on objectives that minimize the total route cost and the number of stops to the same client and included a workload balance component between vehicles for each period. This component of the objective minimizes the maximum service time over the weeks of the planning horizon and the days of the week. Furthermore, they imposed a maximum route duration for each vehicle. A decomposition approach was used first to assign clients to weeks, and then to assign them to a day within the selected week. A variant of the classical VRP was finally solved for each day using a three-phase adaptive large-neighborhood search. This algorithm was applied in a case study for a hygiene service company performing more than 69 951 visits for 6000 clients over 12 weeks, leading to significant practical savings.

Liu et al. (2013) studied a PVRP for home health care in which three types of patients require services. The primary objective was to reach equity by minimizing the maximum route time for all vehicle routes over the week. The solution method combined tabu search and local search. Liu et al. (2020) balanced the workload among routes for a periodic home health care assignment problem by partitioning the service area into regions. Finally, Schönberger (2016) set upper bounds on route duration to ensure daily workload balance in a multi-period VRP.

Ribeiro and Lourenço (2001) was, to our knowledge, one of the very few studies considering a multi-period VRP in which equity is evaluated over multiple periods. The problem is solved with a weighted sum objective considering cost, equity, and market share. The equity objective minimizes the standard deviation of the workloads over many periods, where the workload corresponds to the demand served by each driver. Small instances were solved with a commercial solver. Huang et al. (2019) studied a PVRP for which the objective function minimizes the total workload of all drivers, and the maximum workload difference between two drivers cannot exceed a threshold. The equity metric, in this case, corresponds to the sum of travel and service times. The results show that workload equity among drivers can be achieved at a reasonable cost. Finally, Mancini et al. (2021) recently considered workload balance and service consistency in a collaborative multi-period VRP, where the number of clients assigned to a carrier over the planning horizon is constrained. They also examined how workload balance and service consistency impact the total solution cost.

As seen in this review, very few studies have focused on multi-period VRPs that account for equity, and even fewer studies have considered equity over multiple periods. The present study fills this critical methodological gap. It proposes a simple solution approach for an equitable vehicle routing problem defined over multiple periods, and analyses the resulting equity depending on the planning horizon.

4.3 Problem Statement

The MVRPB can be formally defined on a complete graph $G = (V,\ E)$, where V is the set of vertices and E is the set of edges. Let $V = \{0\} \cup C$, where C is a set of vertices representing clients and vertex 0 is the depot. The cost d_{ij} corresponds to the length of edge $(i, j) \in E$ representing a direct trip from i to j. For simplicity, we assume that all distances d_{ij} are integer. Each client $i \in C$ is characterized by a list of visit days (i.e., periods) within a planning horizon of T periods and by a demand quantity q_i^t on each of these days (i.e., demand quantities can differ between days). Finally, m drivers are available through the planning horizon to perform the visits, and we are given an unlimited fleet of homogeneous vehicles with capacity Q located at the depot.

Solving the MVRPB amounts to finding routes for each day in such a way that (i) each client is visited on each requested day, (ii) each route for a given day is assigned to a single driver, (iii) no driver operates more than one route in a day, and (iv) no route exceeds the vehicle capacity. Note that drivers may not necessarily work during each day of the planning horizon. The objectives are to optimize distance and balance the drivers' workloads over the planning horizon.

Matl et al. (2018) discussed different *equity metrics* (e.g., travel time, distance, demand quantity served, number of clients) and *equity measures* (e.g., the maximum workload of a driver, the difference between the smallest and largest workloads, and the standard deviation of workloads). In this work, we focus on distance-based workload equity among drivers. Therefore, the workload of a driver is the total distance traveled by the driver over the planning horizon. We use the Min-Max equity measure, which aims to achieve equity by minimizing the maximum total distance of the routes traveled by any driver over the planning horizon.

4.4 Solution Approach

To solve the MVRPB, we design a two-phase solution approach. As seen in the remainder of this section, our approach follows a hierarchical objective: it first guarantees a routing solution with minimum cost (i.e., with minimum total distance over the planning horizon) and then maximizes workload balance. This permits us to evaluate the extent to which workload equity can be ensured over a longer planning horizon without sacrificing economic efficiency. Moreover, as seen in the following, the optimal routes found in the first phase will permit us to calculate a bound on the best possible workload equity for any solution.

Our method unfolds in two stages. Firstly, it solves a CVRP for each period considering only the deliveries of this period and minimizing cost (total distance). Then, it solves an allocation problem to assign routes to drivers on each period, intending to optimize workload balance. The remainder of this section details the techniques designed to perform each step efficiently.

4.4.1 First Phase – Distance Optimization

The first phase consists of solving one CVRP per period $t \in T$ to obtain high-quality routing plans minimizing distance. We rely on mixed-integer linear programming (MILP) techniques to solve these problems. We first present a compact mathematical formulation of the problem and then discuss its solution by branch-and-price.

Table 4.1 summarizes the notations used in the mathematical models. For each period t, the resulting CVRP$_t$ subproblem can be mathematically formulated using a three-index undi-

Table 4.1 Notations used in the mathematical programs

Sets	C	Set of clients
	E	Set of edges
	V	Set of nodes, $V = C \cup \{0\}$
	C_t	Set of clients in period $t \in T$
	E_t	Set of edges in period $t \in T$
	V_t	Set of vertices in period $t \in T$, $V_t = C_t \cup \{0\}$
	R_t	Set of routes in period $t \in T$
Parameters	T	Number of time periods
	m	Number of drivers (i.e., maximum number of vehicle routes in each period)
	Q	Capacity of each vehicle
	q_i^t	Demand of client $i \in C_t$ in period $t \in T$
	d_{ij}	Distance of edge $(i,j) \in E$
Variables	y_{ir}	Binary variable equal to 1 if client i is on route r, $i \in C$, $r \in R_t$.
	x_{ijr}	Binary variable equal to 1 if edge $(i,j) \in E$ is on route r, $i,j \in V$, $r \in R_t$.

rected formulation, as in Baldacci et al. (2010). In this formulation, the cut set for any $S \subseteq C_t$ is defined as $\delta(S) = \{(i,j) \in E_t : i \in S,\, j \notin S \text{ or } i \notin S,\, j \in S\}$:

$$(CVRP_t) \qquad \min \sum_{r=1}^{m} \sum_{(i,j)\in E_t} d_{ij} x_{ijr} \tag{4.1}$$

$$\text{s.t.} \quad \sum_{r=1}^{m} y_{ir} = 1 \qquad i \in C_t \tag{4.2}$$

$$\sum_{i\in C_t} q_i^t y_{ir} \le Q \qquad r \in \{1,\dots,m\} \tag{4.3}$$

$$\sum_{(i,j)\in\delta(\{i\})} x_{ijr} = 2y_{ir} \qquad i \in C_t, r \in \{1,\dots,m\} \tag{4.4}$$

$$\sum_{(i,j)\in\delta(S)} x_{ijr} \ge y_{ir} \qquad S \subseteq C_t \,:\, |S| \ge 2, i \in S, r \in \{1,\dots,m\} \tag{4.5}$$

$$x_{ijr} \in \{0,1\} \qquad (i,j) \in E_t \backslash \{(0,j) : j \in C_t\}, r \in \{1,\dots,m\} \tag{4.6}$$

$$x_{0jr} \in \{0,1,2\} \qquad j \in C_t, r \in \{1,\dots,m\} \tag{4.7}$$

$$y_{ir} \in \{0,1\} \qquad i \in C_t, r \in \{1,\dots,m\} \tag{4.8}$$

Objective (4.1) minimizes the total routing cost. Constraints (4.2) force each client to be served by exactly one route. Constraints (4.3) ensure that capacity constraints are respected. Constraints (4.4) are flow-conservation constraints for the routes that also link the x and y variables. Constraints (4.5) are the sub-tour-elimination constraints. These constraints guarantee that each route contains the depot.

Previous studies on integer programming approaches for the CVRP indicate that directly solving Model (4.1–4.8) is ineffective for medium instances with more than a few dozen clients and that combining cuts and column generation generally provides better results. Accordingly, we exploit the VRPSolver framework (Pessoa et al., 2020) for an efficient solution to this problem. This solver provides a generic branch-and-cut-and-price (BCP) framework for many classes of MILPs, including, among others, the considered problem setting. It has achieved a competitive or superior performance on standard test instances when compared with specialized VRP algorithms and is currently available at `https://vrpsolver.math.u-bordeaux.fr/`.

This algorithmic framework relies on the solution of successive pricing subproblems that take the form of resource-constrained shortest paths (RCSPs) on a path-generator graph (VRPGraph). A bidirectional-labeling dynamic programming algorithm is then used to solve the RCSPs. VRPSolver relies on the concept of packing sets to generalize well-known cuts. Essentially, a packing set is a subset of arcs such that at most one arc from the given subset appears in the paths that are part of an optimal solution. Packing sets are defined in accordance with the application considered and the associated model. In our specific case, the packing sets represent limited memory rank-1 cuts (a generalization of the subset row cuts – Jepsen et al. 2008) and rounded capacity cuts (Laporte and Nobert, 1983). The branching rule in VRPSolver is based on accumulated resource consumption and, if needed to enforce integrality, on a generalization of the branching rule of Ryan and Foster (1981).

Finally, the performance of VRPSolver depends on the availability of a good initial upper bound (UB) to limit the search. To find such an initial solution and bound, we use the hybrid genetic search (HGS – Vidal et al. 2012; Vidal 2022), a state-of-the-art metaheuristic for the CVRP. HGS could be used as a stand-alone approach for the first phase in time-critical applications, or if the scale of the problems becomes too large for an exact solution. Still, we opted to additionally rely on the exact algorithm in this phase, as this will allow us to obtain lower bounds on the best achievable workload equity (see next section).

4.4.2 Second Phase – Equitable Workload Allocation

The second phase of the algorithm takes as input the solution R^* found in the previous phase, represented as the set of optimal routes R_t^* for each period. It seeks to achieve a fair distribution of these routes among m identical drivers. In this stage, we use a Min-Max objective to minimize the maximum workload of any driver (i.e., the maximum total distance driven by a driver over all periods).

MILP formulation and bounds

This allocation problem can also be mathematically formulated as a MILP. Let d_r represent the distance driven on each route $r \in R_t^*$ found in the first phase, and let z_{rk}^t be a binary variable equal to 1 if route r is assigned to driver k in time period t. Finally, let Δ be a continuous variable capturing the maximum distance for a driver over the planning horizon. The best possible workload equity for the considered set of routes can be found by solving the following model:

$$\min \ \Delta \tag{4.9}$$

$$\text{s.t.} \sum_{t=1}^{T} \sum_{r \in R_t^*} d_r z_{rk}^t \leq \Delta \qquad k \in \{1, \dots, m\} \tag{4.10}$$

$$\sum_{r \in R_t^*} z_{rk}^t \leq 1 \qquad t \in \{1, \dots, T\}, k \in \{1, \dots, m\} \tag{4.11}$$

$$\sum_{k=1}^{m} z_{rk}^t = 1 \qquad t \in \{1, \dots, T\}, r \in R_t^* \tag{4.12}$$

$$z_{rk}^t \in \{0, 1\} \qquad t \in \{1, \dots, T\}, r \in R_t^*, k \in \{1, \dots, m\} \tag{4.13}$$

$$\Delta \in \mathbb{R}^+. \tag{4.14}$$

Objective (4.9) and Constraints (4.10) model the minimization of the maximum workload over all drivers. Constraints (4.11) ensure that each driver serves at most one route in each period, whereas Constraints (4.12) ensure that each route is assigned to exactly one driver in each period. Finally, Constraints (4.13) and (4.14) define the domain of the decision variables. Note that the inequality in Constraints (4.11) can be replaced by an equality if the number of drivers matches the number of routes found in the first phase in any given period t. The resulting formulation can be viewed as a variant of the bin packing problem (BPP) with conflicts (Capua et al., 2018).

Let $\Delta_{\text{OPT}}(R)$ be the optimal workload produced by solving the allocation problem for a given first-phase routing solution R, and let $D(R)$ be the total distance of all routes of a solution R over all periods. The following bounds are valid:

Property 1 *The best possible workload allocation for a routing solution R is such that*

$$\Delta_{\text{OPT}}(R) \geq \left\lceil \frac{D(R)}{m} \right\rceil. \tag{4.15}$$

Proof. The proof directly derives from the formulation of the assignment problem. Summing

Constraint (4.10) over $k \in \{1, \ldots, m\}$ gives:

$$\sum_{k=1}^{m} \sum_{t=1}^{T} \sum_{r \in R_t} d_r z_{rk}^t \leq m\Delta_{\text{OPT}}(R). \tag{4.16}$$

Next, using Equation (4.12) leads to:

$$\sum_{t=1}^{T} \sum_{r \in R_t} d_r \leq m\Delta_{\text{OPT}}(R) \Longrightarrow \Delta_{\text{OPT}}(R) \geq \frac{D(R)}{m} \tag{4.17}$$

Finally, since the distances d_{ij} are integer, then $\Delta_{\text{OPT}}(R)$ is also an integer, and the right-hand side of the inequality can be rounded up, giving the announced result.

Property 2 *Let Δ_{OPT} be the best possible workload equity achievable in any solution of the MVRPB (including first-stage solutions that are not optimal in terms of distance), then:*

$$\Delta_{\text{OPT}} \geq \left\lceil \frac{D(R^*)}{m} \right\rceil. \tag{4.18}$$

Proof. As a consequence of Property 1, the best possible workload equity over all possible routing solutions R satisfies the following relation:

$$\Delta_{\text{OPT}} = \min_{R} \Delta_{\text{OPT}}(R) \geq \min_{R} \left\lceil \frac{D(R)}{m} \right\rceil = \left\lceil \frac{\min_R D(R)}{m} \right\rceil = \left\lceil \frac{D(R^*)}{m} \right\rceil. \tag{4.19}$$

This relation gives us a lower bound LB $= \lceil D(R^*)/m \rceil$, which permits us to evaluate how far our MVRPB solutions are from the best possible workload equity, calculated by assuming that the total amount of workload from distance-optimal routing solutions is evenly distributed among drivers. It is important to remark that Δ_{OPT} can be smaller than $\Delta_{\text{OPT}}(R^*)$ since the best workload balance over multiple periods can involve routes that do not belong to any optimal CVRP solution of a given period. In contrast, the bound announced in Property 2 holds for any MVRPB solution.

Set-partitioning reformulation and solution approach

A direct solution of Formulation (4.9–4.14) using standard MILP solvers such as CPLEX is ineffective. This is partly due to symmetry, given that all drivers are considered identical, and renumbering them produces equivalent solutions. One way to circumvent this issue is to solve this problem as a sequence of set-partitioning feasibility problems for different values of Δ as defined in Model (4.20–4.22).

$$\sum_{\sigma \in \Omega_\Delta} \lambda_\sigma = m \tag{4.20}$$

$$\sum_{\sigma \in \Omega_\Delta} a_{\sigma r} \lambda_\sigma = 1 \qquad t \in \{1, \ldots, T\}, r \in R_t^* \tag{4.21}$$

$$\lambda_\sigma \in \{0, 1\} \qquad \sigma \in \Omega_\Delta \tag{4.22}$$

In this formulation, each element $\sigma \in \Omega_\Delta$ represents an admissible combination of routes (i.e., a schedule) that a driver can operate over the planning horizon without exceeding a workload of Δ. Each constant $a_{\sigma r}$ takes value 1 if and only if r belongs to σ. Each binary variable λ_σ takes value 1 if this schedule is selected for one driver. Constraints (4.20) ensure that work schedules are created for the m drivers, and Constraints (4.21) ensure that each route appears exactly in one schedule.

Binary search strategy. Finding a feasible solution of Model (4.20–4.22) for a given Δ means that there exists a feasible allocation of the routes to drivers in such a way that the maximum workload over the planning horizon does not exceed Δ. Therefore, the optimal workload balance is such that $\Delta_{\text{OPT}}(R^*) \leq \Delta$. In contrast, proving that this model is infeasible would imply that $\Delta_{\text{OPT}}(R^*) > \Delta$.

With this, we develop a strategy based on a binary search to locate $\Delta_{\text{OPT}}(R^*)$. Initially, we start with $\Delta = LB = \left\lceil \frac{D(R^*)}{m} \right\rceil$ and solve Model (4.20–4.22). If this model is feasible, then we have attained the best possible workload equity. Otherwise, this means there is no solution with a workload balance of $\Delta = \left\lceil \frac{D(R^*)}{m} \right\rceil$. In this case, we use a **construction heuristic** to find an initial feasible solution and therefore an upper bound (UB) value for Δ. This heuristic consists in ordering the items (i.e., all the routes over the planning horizon) by decreasing workload (distance) and then following this order to insert them iteratively into a compatible bin (i.e., a driver that does not operate a route on that day) that has the current smallest workload. At this point, we have a range for the optimum (integer) value and locate it by binary search, using Model (4.20–4.22) to determine feasibility at each step. This process stops when the model for $\Delta_{\text{OPT}}(R^*) - 1$ is infeasible and the model for $\Delta_{\text{OPT}}(R^*)$ is feasible.

Solution of each subproblem. Model (4.20–4.22) contains an exponential number of variables $\sigma \in \Omega_\Delta$, therefore a direct solution approach is impractical. To solve this problem, we rely once again on the branch-and-price framework provided by VRPSolver. Pessoa et al. (2021) provides adaptations of VRPSolver to the classical BPP and other variants such as vector packing, variable-sized BPP, and variable-sized BPP with optional items. To solve our problem with VRPSolver, we essentially need to redefine the path-generator graph

(VRPGraph).

Figure 4.1 represents the VRPGraph for the classical BPP, assuming that I items need to be packed. Each node in this graph corresponds to one item, except node v_0, representing a starting point. Item i of weight w_i is loaded in the bin each time we use arc a_{i+} along the path from the start node to the end node (conversely, item i is not loaded in the bin if arc a_{i-} is used). Next, each path generated in this graph that does not exceed the bin capacity defines a new packing (column) for the column-generation algorithm. Capacity is the only resource consumed in VRPGraph, and a packing set is made of a subset of arcs with nonzero consumption of the resource.

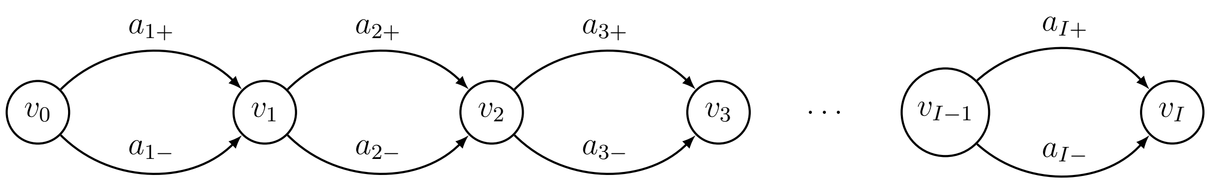

Figure 4.1 Path-generator graph for the BPP.

Figure 4.2 provides an adapted path-generator graph for Problem (4.20–4.22). In this graph, nodes correspond to periods, except the first node P_0, which represents a starting point. An arc of type a_{ij+} then goes from P_{i-1} to P_i for each possible route $j \in \{1, 2, 3, \ldots, n_i\}$ in period P_i, where n_i is the number of routes in period P_i. If one of these parallel arcs is used, the corresponding route is assigned to the driver and contributes to the driver's total workload (total distance limited to Δ, which stands as the bin's capacity). An arc of type a_{i-} is also available to represent the possibility of assigning no route to the driver in period P_i. With these conventions, a path from the start node to the end node that does not exceed the workload limit Δ corresponds to a feasible assignment of routes to a driver over the planning horizon.

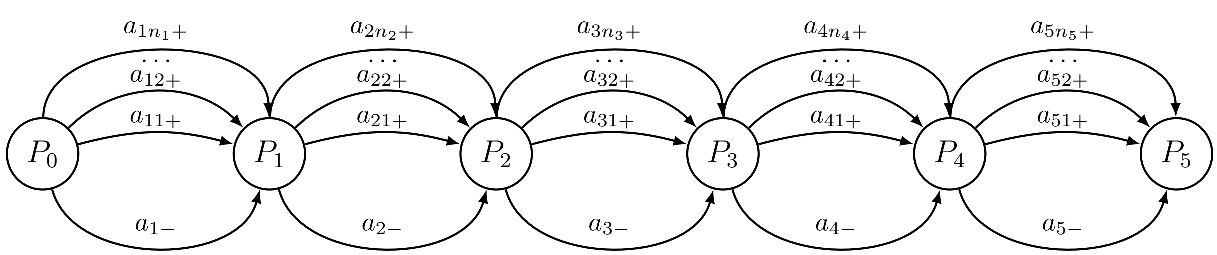

Figure 4.2 Path-generator graph for the second-stage model.

We use the standard parameter setting of VRPSolver with just a minor modification to its diving heuristic. VRPSolver typically uses a diving heuristic before branching to improve the primal solution (Sadykov et al., 2019), but only at the root node. In contrast, our implementation allows strong diving at each node to quickly locate feasible solutions. If such a solution is found, then the solver can be immediately stopped since the model is known to be feasible.

In general, VRP solver uses heuristic pricing during diving. When we use strong diving at each node, VRPSolver would be able to confirm that the feasibility problem (BPP) is feasible in a few seconds, by showing that the LB is equal to the upper bound (number of available bins).

4.5 Computational Study

The goal of our experimental study is twofold: (i) evaluating the performance of the proposed solution approach and especially the computational effort needed for each of its steps, and (ii) measuring to which extent workload equity can be achieved over multiple periods without sacrificing economic efficiency, by simply allocating cost-optimal routes to drivers in an equitable fashion as done in our two-phases approach.

Our experiments are conducted on a 2.4 GHz Intel Gold 6148 Skylake processor with 8 GB of RAM. The VRPSolver interface is implemented in Julia v1.4.2 with JuMP v0.18. VRPSolver uses BaPCod, a C++ library for implementing a generic BCP, and CPLEX 12.8 to solve the linear and mixed-integer linear programs. All experiments have been conducted on a single thread.

4.5.1 Test Instances

To construct test instances for the MVRPB, we rely on a subset of the CVRP instances of Uchoa et al. (2017), as they include diverse characteristics: distribution and number of clients, depots locations, and average route length. The complete set contains 100 Euclidean instances with 100 to 1000 clients. The distances are rounded to the nearest integer as in the original instances.

Since the MVRPB is defined in a multi-period context, we had to modify the original instances. Therefore, we selected the instances X-n200-k36, X-n204-k19, X-n209-k16, X-n214-k11, X-n219-k73, X-n223-k34, X-n228-k23, X-n233-k16, X-n237-k14, and X-n242-k48 including between 199 and 241 clients from Uchoa et al. (2017). We generated three different 10-period MVRPB configurations for each of these ten instances by randomly selecting 50, 75, or 100 clients in each period. Finally, for each period and client with demand d in the original instance, we randomly selected a new demand realization from a uniform integer distribution in $\{\lceil 0.5 \times d \rceil \ldots, \lceil 1.5 \times d \rceil\}$. This way, clients can have different demands at different periods. We repeated this generation (customers and demands selection) ten times for each configuration, leading to $10 \times 3 \times 10 = 300$ MVRPB instances defined over 10 periods. Finally, to obtain instances with fewer periods $T \in \{2, 3, 5, 7\}$, we retained the first T periods of each 10-period instance. The number of drivers in each 10-period instance was set to the maximum number of routes from optimal CVRP solutions over the ten periods. Consequently, some drivers may be idle for a given period. The number of drivers in the 2-, 3-, 5-, and 7-period instances is kept identical to the number of drivers in the corresponding 10-period instance.

4.5.2 Computational Performance

We first evaluate the performance of each of the two steps of the proposed approach: the computational effort needed to find optimal CVRP solutions in each period, and the effort to find an equitable workload allocation in the second step. We refer to these steps as (1) route optimization and (2) multi-period workload balancing.

Route optimization. Tables 4.2 to 4.4 report the performance of the route-optimization step for the 10-period instances with 50, 75, and 100 clients in each period. For brevity, the results are presented in aggregated form, with one line for each original instance of Uchoa et al. (2017), by averaging over the 10 corresponding MVRPB instances and 10 periods. From left to right, the columns report the names of the associated original instances, the average traveled distance per period in the solutions found by HGS and VRPSolver, the average computational time of these two methods, and finally, the number $\#k$ of drivers.

Table 4.2 Performance of the route-optimization step, for MVRPB instances with 50 clients per period

Instance	Distance		T(s)		#k
	HGS	VRPSolver	HGS	VRPSolver	
X-n200-k36	16284.45	16281.48	15.8	614.1	10.0
X-n204-k19	7200.85	7200.85	14.0	7.8	5.3
X-n209-k16	9699.69	9699.69	14.2	14.2	4.8
X-n214-k11	3782.88	3782.88	14.2	52.1	3.4
X-n219-k73	28562.15	28562.15	13.4	2.3	17.0
X-n223-k34	11464.69	11464.17	13.1	7.3	9.5
X-n228-k23	7652.85	7652.72	14.1	25.5	7.3
X-n233-k16	6704.64	6704.64	12.8	12.9	4.5
X-n237-k14	7980.45	7980.45	13.1	31.4	3.0
X-n242-k48	19970.99	19970.05	15.0	7.6	12.1

Table 4.3 Performance of the route-optimization step, for MVRPB instances with 75 clients per period

Instance	Distance		T(s)		#k
	HGS	VRPSolver	HGS	VRPSolver	
X-n200-k36	23388.65	23379.47	28.7	185.5	14.7
X-n204-k19	9294.97	9294.97	22.0	31.6	7.9
X-n209-k16	13272.78	13272.68	24.5	60.1	6.6
X-n214-k11	4881.22	4881.20	25.2	268.2	5.0
X-n219-k73	41826.76	41826.76	20.3	2.8	25.0
X-n223-k34	16035.39	16034.47	20.8	23.4	13.3
X-n228-k23	10379.97	10379.84	21.8	239.4	10.2
X-n233-k16	8480.25	8479.83	19.6	253.3	6.3
X-n237-k14	10870.43	10870.43	21.7	34.9	5.0
X-n242-k48	28644.96	28640.61	27.3	123.6	17.2

Table 4.4 Performance of the route-optimization step, for MVRPB instances with 100 clients per period

Instance	Distance		T(s)		#k
	HGS	VRPSolver	HGS	VRPSolver	
X-n200-k36	30780.50	30758.06	47.6	979.1	19.6
X-n204-k19	11535.58	11534.75	32.5	470.4	10.0
X-n209-k16	16724.49	16722.15	39.9	420.6	8.3
X-n214-k11	6076.82	6076.64	39.2	1041.6	6.1
X-n219-k73	55473.58	55473.58	27.6	2.7	34.0
X-n223-k34	20522.69	20519.90	31.8	69.3	17.1
X-n228-k23	12888.06	12887.60	32.2	950.6	12.5
X-n233-k16	10288.46	10287.92	27.0	1296.1	8.0
X-n237-k14	13518.71	13518.58	35.0	697.4	6.0
X-n242-k48	37262.13	37247.82	40.9	324.4	22.4

As seen in these experiments, the computational time used by VRPSolver to optimally solve the underlying CVRP problem for each period is generally small, with a median value of 25.0 seconds. However, on a handful of cases, the computational time may be long, reaching 12.8 hours in the worst case (one exceptional case in 3 000 CVRP single-period sub-problems). In contrast, HGS has a more controllable computational time, ranging from 11.4 seconds to 2.43 minutes, with a median value of 21.8 seconds. We observe that the initial solutions found by HGS were almost optimal in terms of their distance, with an average distance over all instances of 16715.0 compared to 16712.9 for VRPSolver, i.e., with an average gap error of only 0.013% from optimal solution values. Given this, we recommend using HGS as the underlying solution approach for the route optimization step in practical time-critical applications. In the context of this study, we decided to complete the solution process to achieve proven optima with VRPSolver, as this will subsequently permit us to derive bounds on the best possible workload balance through Equation (4.18).

Finally, we must observe that the problems associated with each period are independent, such that it is possible to solve them in parallel. We used this observation in our experiments, as the multi-core structure of our processor permitted us to independently solve the CVRPs associated with each period on a different core, therefore maximizing our utilization of available computational resources and reducing the total time needed to conduct our experiments.

Multi-period workload balancing. In the workload balancing step, the routes of the optimal CVRP solution for each period are assigned to drivers to minimize the maximum (Min-Max) total distance traveled by each driver over the entire planning horizon. We build our analysis on three key workload measurements:

- UB – The initial workload produced by the constructive approach described in Section 4.4.2. The workload corresponds to the largest total distance for a driver over the entire planning horizon.
- Opt – The optimal workload obtained after completing the binary search.
- LB – The lower bound of Equation (4.18), which assumed that distance is optimal and workload equity is perfect (often this does not match a practical solution).

Tables 4.5 to 4.7 report the workload values of UB, Opt, and LB for the different instances, with a varying number of periods and with 50, 75, and 100 clients in each period, respectively. Each line in the tables corresponds to an average value over ten different MVRPB instances. These tables also indicate the average number of binary-search operations in our algorithm and the average solution time.

Table 4.5 Performance of the multi-period workload balancing step – MVRPB instances with 50 clients per period

Tests	2 periods					3 periods					5 periods					7 periods					10 periods				
	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)
X-n200-k36	3199.6	3551.5	3551.5	10.0	6.46	4858.2	5291.1	5033.9	9.7	7.18	8084.7	8630.9	8090.3	10.0	56.35	11372.8	11989.7	11372.8	1.0	8.47	16274.3	16907.1	16274.5	3.0	279.05
X-n204-k19	2735.3	3185.4	3185.4	10.1	6.54	4107.5	4570.4	4320.7	9.8	6.80	6810.6	7095.7	6822.5	9.1	5.82	9584.2	9721.3	9585.0	7.0	18.75	13663.1	13883.5	13663.1	1.0	9.39
X-n209-k16	4077.9	4846.0	4846.0	11.0	6.65	6167.3	7088.4	6421.1	10.9	7.36	10276.2	10574.6	10304.9	9.1	5.78	14297.5	14690.3	14300.5	9.4	23.59	20353.8	20725.2	20353.8	1.0	8.15
X-n214-k11	2256.0	2644.0	2644.0	10.1	6.39	3403.7	3643.4	3546.0	8.9	5.98	5608.7	5886.0	5652.8	9.2	5.89	7939.2	8186.8	7946.5	8.7	14.93	11333.4	11498.9	11333.5	1.6	8.62
X-n219-k73	3378.8	3696.6	3696.6	9.9	6.68	5012.3	5469.3	5035.8	9.9	6.03	8377.7	8650.5	8377.7	1.0	740.10	11778.3	12115.5	11778.3	2.2	17.72	16801.8	17059.7	16801.8	1.0	23.96
X-n223-k34	2436.9	2909.9	2909.9	10.3	6.19	3630.5	4052.6	3708.1	9.6	5.34	5991.6	6285.4	5994.4	9.3	49.34	8438.0	8702.3	8438.0	1.0	7.77	12101.1	12276.0	12101.2	1.7	439.17
X-n228-k23	2084.1	2705.4	2705.4	10.7	6.30	3156.1	3665.2	3412.0	10.0	4.98	5254.3	5650.0	5271.0	9.3	12.61	7291.4	7592.9	7291.5	1.9	11.91	10513.6	10815.5	10513.6	1.3	9.87
X-n233-k16	2998.2	3541.6	3541.6	10.5	6.73	4524.8	5116.2	4785.1	9.9	6.18	7560.4	8115.5	7594.1	9.7	6.62	10527.6	11028.3	10530.8	9.8	20.70	15072.1	15367.5	15072.1	1.0	7.69
X-n237-k14	5336.4	5718.7	5718.7	9.7	6.39	7995.7	8451.0	8196.9	9.9	5.63	13326.9	13647.9	13384.9	8.4	6.71	18636.1	18856.6	18646.0	8.2	8.15	26601.9	26985.1	26602.1	2.6	8.58
X-n242-k48	3329.5	3875.0	3875.0	10.6	6.71	4957.2	5583.6	5063.6	10.2	5.53	8224.7	8669.3	8226.3	9.7	52.25	11365.1	11899.5	11509.3	2.1	10.54	16527.5	16799.3	16527.5	1.0	18.65
Ave.	3183.3	3667.4	3667.4	10.3	6.50	4781.3	5293.1	4952.3	9.9	6.10	7951.6	8320.6	7971.9	8.5	94.15	11123.0	11478.3	11139.9	5.1	14.25	15924.3	16231.8	15924.3	1.5	81.31

Table 4.6 Performance of the multi-period workload balancing step – MVRPB instances with 75 clients per period

Tests	2 periods					3 periods					5 periods					7 periods					10 periods				
	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)
X-n200-k36	3201.3	3463.1	3463.1	9.6	6.67	4788.9	5201.0	4916.3	9.8	7.60	7954.8	8459.4	7955.3	6.0	23.64	11132.6	11608.2	11132.6	1.0	11.97	15919.8	16472.8	15919.8	1.0	20.89
X-n204-k19	2345.8	2695.6	2695.6	9.8	6.78	3540.7	3841.5	3622.5	9.4	6.78	5895.7	6219.6	5898.7	9.6	27.47	8257.0	8633.6	8257.0	1.0	7.93	11786.4	11941.7	11786.4	1.0	13.86
X-n209-k16	4089.0	4606.9	4606.9	10.4	6.62	6136.5	6724.1	6284.7	10.1	6.68	10188.3	10686.7	10196.7	9.9	16.26	14216.4	14493.9	14216.8	4.2	22.23	20224.5	20496.7	20224.5	1.0	15.97
X-n214-k11	1951.2	2292.2	2292.2	9.7	6.29	2918.5	3289.1	3090.3	9.7	6.40	4846.5	5115.8	4859.4	9.1	8.21	6805.0	7026.2	6805.5	5.1	13.41	9762.8	9945.0	9762.8	1.0	9.21
X-n219-k73	3349.5	3604.7	3604.7	9.6	7.28	5002.9	5498.0	5014.2	9.7	11.77	8337.8	8553.9	8337.9	1.8	1705.40	11696.1	11979.5	11696.1	1.0	20.36	16731.2	17034.0	16731.3	1.9	52.37
X-n223-k34	2424.1	2770.6	2770.6	9.9	6.71	3604.4	3978.5	3653.8	9.5	7.57	5983.5	6219.6	5984.0	5.2	132.47	8404.2	8601.1	8404.2	1.0	10.84	12071.3	12249.7	12071.3	1.0	22.66
X-n228-k23	2033.6	2466.4	2462.6	10.0	6.82	3053.9	3622.9	3188.7	10.1	7.28	5017.0	5348.4	5020.6	9.7	51.00	7103.5	7431.2	7103.5	1.0	7.72	10217.8	10484.5	10217.8	1.0	17.36
X-n233-k16	2709.8	3103.9	3103.9	10.0	6.62	4055.7	4570.3	4251.0	9.9	7.38	6818.4	7142.7	6827.9	9.5	11.17	9532.3	9847.3	9532.4	1.9	11.33	13525.2	13792.1	13525.2	1.0	13.70
X-n237-k14	4313.7	5148.3	5148.3	11.1	7.14	6472.5	7475.0	6965.4	10.9	7.04	10835.0	11452.6	10854.5	9.2	7.68	15187.6	15520.8	15189.3	9.4	20.20	21741.2	21826.9	21741.2	1.0	8.68
X-n242-k48	3411.4	3886.9	3886.9	10.4	6.97	5107.4	5707.5	5171.3	10.1	8.16	8470.0	8984.9	8470.2	2.8	194.67	11752.5	12041.8	11752.5	1.0	12.82	16658.9	16882.6	16658.9	1.0	22.91
Ave.	2982.9	3403.9	3403.5	10.1	6.79	4468.1	4990.8	4615.8	9.9	7.67	7434.7	7818.4	7440.5	7.3	217.80	10408.7	10718.4	10409.0	2.7	13.88	14863.9	15112.6	14863.9	1.1	19.76

Table 4.7 Performance of the multi-period workload balancing step – MVRPB instances with 100 clients per period

Tests	2 periods					3 periods					5 periods					7 periods					10 periods				
	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)	LB	UB	Opt	#It	T(s)
X-n200-k36	3161.3	3439.1	3407.4	9.4	7.61	4727.0	5311.2	4834.0	10.0	8.86	7865.5	8477.5	7865.5	1.0	76.79	10982.5	11470.7	10982.5	1.0	13.99	15703.2	16244.7	15703.2	1.0	25.56
X-n204-k19	2309.4	2607.1	2607.1	9.6	7.21	3460.6	3731.8	3507.1	9.1	7.99	5763.0	6048.3	5764.1	9.5	90.53	8071.2	8337.0	8071.2	1.0	10.42	11527.4	11775.4	11527.4	1.0	12.66
X-n209-k16	4043.6	4448.2	4448.2	10.1	8.00	6062.4	6610.1	6173.2	10.1	6.33	10137.2	10570.4	10140.8	9.8	35.44	14176.1	14523.5	14176.1	1.0	8.28	20204.7	20512.9	20204.7	1.0	11.06
X-n214-k11	1969.8	2176.2	2176.2	9.2	7.96	2967.3	3311.4	3053.0	9.2	5.91	4987.2	5239.6	4994.1	8.9	10.83	7026.2	7299.6	7026.4	2.5	10.80	9986.3	10105.7	9986.3	1.0	9.27
X-n219-k73	3239.3	3486.3	3486.3	9.5	7.36	4879.9	5296.2	4886.1	9.8	21.87	8159.9	8477.6	8159.9	1.0	15.04	11397.0	11765.1	11397.0	1.0	31.32	16316.2	16654.4	16316.2	1.0	102.17
X-n223-k34	2384.1	2708.7	2708.7	9.7	6.95	3603.1	3871.1	3631.7	9.1	9.55	6054.6	6243.3	6054.6	1.0	433.94	8413.6	8631.7	8413.6	1.0	12.68	12001.9	12232.6	12001.9	1.0	20.40
X-n228-k23	2012.2	2479.0	2479.0	10.3	7.32	3064.6	3598.2	3180.6	10.0	8.19	5112.6	5417.1	5114.5	7.4	29.32	7169.0	7410.0	7169.0	1.0	9.67	10325.3	10654.1	10325.3	1.0	12.57
X-n233-k16	2524.4	2955.2	2923.2	10.0	7.58	3754.3	4308.4	3885.5	10.0	7.79	6307.1	6874.5	6310.2	9.9	28.01	8849.7	9410.8	8849.7	1.0	11.70	12602.4	12996.6	12602.4	1.0	10.34
X-n237-k14	4487.4	4942.2	4942.2	10.3	7.41	6733.5	7343.4	6919.2	10.3	7.22	11209.6	11801.3	11218.6	10.1	11.40	15710.0	15927.3	15710.3	3.4	14.09	22531.5	22764.5	22531.5	1.0	9.66
X-n242-k48	3386.4	3832.8	3832.8	10.3	7.17	5049.7	5545.3	5072.4	9.8	10.15	8254.0	8804.4	8254.0	1.0	1467.53	11637.7	12000.6	11637.9	2.9	552.76	16633.5	16834.3	16633.5	1.0	32.24
Ave.	2951.8	3307.5	3301.1	9.8	7.46	4430.2	4892.7	4514.3	9.7	9.39	7385.1	7795.4	7387.6	6.0	219.88	10343.3	10677.6	10343.4	1.6	67.57	14783.2	15077.5	14783.2	1.0	24.59

As seen in these experiments, only a few seconds are required in most cases to complete an optimal multi-period workload balancing step. This confirms the efficiency of our two-stage solution approach. Generally, instances involving a larger number of routes and drivers (e.g., X-n219-k73) lead to more complex workload balancing problems. Again, there is also inherent variability due to the exact solution process, given that MILP approaches can exhibit substantially different computation times when solving instances of similar sizes. Overall, the computational time of the second step ranges from 4.40 to 7600.99 seconds with a median value of 8.53 seconds.

The workload allocation created by the initial construction approach (described in Section 4.4.2) is optimal (i.e., UB = Opt) for 298 out of 300 MVRPB instances with 2 periods, as well as for 15 out of 300 MVRPB instances with 3 periods. In contrast, as soon as the number of periods becomes greater than five, the initial construction approach is unlikely to lead to the best possible workload allocation, and the underlying mathematical model produces much better solutions.

When the planning horizon contains five days or more, we frequently notice cases of "perfect" workload equity, where the obtained workload balance matches the theoretical lower bound (i.e., Opt = LB). The ability to achieve perfect balance comes as a consequence of the increased number of possible assignment solutions, which grows exponentially with the number of periods. In practice, in cases with five periods or more, it is sufficient to focus on cost-optimal routing solutions and to create equitable workloads by careful assignment. Finally, as our binary search strategy includes a first step to verify if a solution with perfect balance exists, all the cases for which perfect balance is possible are solved in a single call to the feasibility subproblem. In the other cases, it generally takes between 7 to 11 steps.

4.5.3 Planning-Horizon Length and Workload Equity

The previous section showed that near-perfect workload equity is achievable, in practice, for planning horizons with at least five periods. To visualize more clearly the impact of the number of periods over workload equity, Figure 4.3 provides additional boxplot representations of the Gap(%) between the ideal workload (LB from Equation (4.18)) and the optimal solution value (Opt) of our two-phases approach. We used the following calculation: Gap = 100 × (Opt − LB)/LB. We provide separate plots for the cases with 50, 75, or 100 customers per period. Each boxplot corresponds to the data of a given planning horizon with $T \in \{2, 3, 5, 7, 10\}$ periods, therefore gathering gap measurements from 100 instances. The whiskers indicate the minimum, first quartile (Q1), median, third quartile (Q3), and maximum. The minimum corresponds to $Q1 - 1.5 \times$ interquartile range, while the maximum corresponds to $Q3 + 1.5 \times$ interquartile range. Outliers that fall beyond the minimum and maximum range are additionally depicted as small circles.

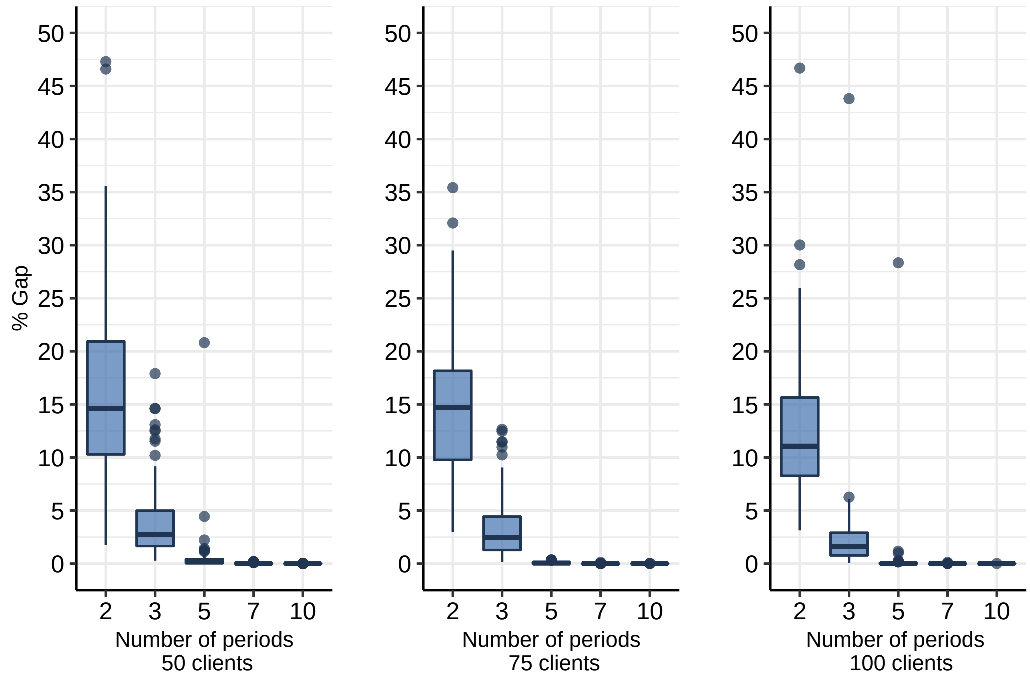

Figure 4.3 Convergence of Opt toward LB as the number of periods increases for instances with 50, 75, and 100 clients in each period.

These boxplots give another viewpoint on the convergence toward the best possible equity as the number of periods increases. In the vast majority of the cases (excluding a few outliers), a planning horizon of five days is sufficient to find equitable solutions with workload discrepancies below 1% between drivers. In these situations, there is no need to seek a trade-off between routing costs and workload equity since optimal routing solutions can be used to achieve equity. Another benefit of our two-stage approach is its flexibility since additional constraints, decisions, and objectives (i.e., routing attributes – Vidal et al. 2013) only need to be integrated with the first phase of the solution approach.

Finally, in the cases with very few periods (e.g., 2 or 3 days), we observe that focusing the search on optimal routing solutions does not permit achieving the best possible workload equity. In such situations, it would be helpful to consider alternative routing solutions. One possibility would consist in producing multiple routing solutions for each period and extending the workload balancing step to include all these alternatives. Another approach, more complex to develop in practice, would be to solve the routing and driver-allocation problem in an integrated manner, considering distance and workload equity in a bi-objective solution method. However, in both cases, the user would need to specify a trade-off between acceptable extra routing costs and the desired workload equity level.

4.6 Conclusions

In this work, we have revisited workload equity in vehicle routing with a longer-term perspective, considering a planning horizon of several days. We have shown that a two-phase optimization approach can identify the most equitable solutions with minimal distance. When the planning horizon exceeds five days, the resulting solutions are optimal in terms of distance and near-optimal (below 1% gap) in terms of equity. Therefore, workload equity appears to be achievable without integrated approaches and trade-off calibration, and without any compromise on operational efficiency.

Several important research perspectives are open in connection with this study. Firstly, our work focused on deterministic settings, where the complete customer demand is known on the planning horizon. Practical situations often involve dynamically-revealed request information, and therefore it is an open question to determine to what extent multi-period workload equity is achievable in dynamic contexts. Another important aspect of practical delivery systems concerns delivery consistency. When the same driver regularly visits the same areas or clients, the service quality and the satisfaction of drivers and clients generally increases (Kovacs et al., 2014). Our approach towards equitable workload allocation largely benefits from the exponential number of possible route-driver allocation combinations. However, consistency may significantly reduce the number of allocation possibilities, such that new approaches may be needed to conciliate three key aspects in a multi-period setting: cost efficiency, workload equity, and delivery consistency.

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CHAPTER 5 GENERAL DISCUSSION

Driver satisfaction in the distribution sector and other related sectors has attracted more attention recently. Gradually, industry owners realize that the satisfaction of their employees is important for the success of their company. Thus, finding a good trade-off between employee satisfaction and routing cost has become an important challenge in this sector.

The literature review indicates that the marginal cost of equity in CVRP, although often reasonable, is not negligible. This fact motivated us to investigate the CVRP over a multi-period planning horizon. Our intuition is that a higher level of equity can be reached at little or no marginal cost in a multi-period context. There are a few studies that consider equal workload objective in a multi-period context and a few of them seek this objective through the whole planning horizon rather than inside each single period.

On this, we defined the MVRPB. The goal of the MVRPB is to create a set of routes for each day such that each client is visited once to serve its demand on each requested day, and each route of each day does not exceed the vehicle capacity. The objectives are to optimize distance and to balance the workload of a driver over the planning horizon.

To this end, a two-phase approach is proposed where efficiency is achieved in the first phase and equity in the second phase. An exact algorithm is used in the first phase to solve the CVRP associated with each period, so that nothing is sacrificed with regard to the routing cost (distance). Then, the second phase allocates the routes in these minimum cost solutions to the drivers with the aim of achieving equity with regard to the total distance traveled by each driver over the whole planning horizon.

It is shown that equity improves with the number of periods increase, thus supporting the hypothesis that it is easier (with less routing cost) to achieve equity over multiple periods than over a single period. So, we do not need to seek a trade-off between routing costs and workload equity since optimal routing solutions can be used to achieve equity. In the cases with very few periods (e.g., 2 or 3 days), we observe that focusing the search on optimal routing solutions does not permit achieving the best possible workload equity. In such situations, it would be helpful to consider alternative routing solutions.

Furthermore, we observed that the solutions are perfectly or quasi-perfectly balanced for a relatively small number of periods (i.e., 7-period and 10-period instances). Therefore, there is no need to resort to sophisticated multi-period problem-solving approaches in such cases to reach equity.

CHAPTER 6 CONCLUSION AND RECOMMENDATIONS

In this thesis, we studied workload equity among drivers for a VRP in a multi-period setting. In the following, we first present a summary of the contributions achieved by this thesis. Then we highlight its limitations and indicate possible future research directions.

6.1 Summary of Work

This thesis studied the MVRPB with the aim of balancing the workload among drivers and quantify its impact on the routing cost. To this end, we first designed a two-phase method for the MVRPB where the first phase focuses on efficiency (cost) and the second phase on workload equity. In Chapter 4, it is shown that equity can be attained at no additional cost when the number of periods is large enough. More precisely, almost perfect equity in the workload of drivers was observed on the instances with a small number of periods (2-, 3- and 5-period instances), while perfect equity was reached on the instances with a larger number of periods (7- and 10- period instances). Thus, the proposed two-phase method is a simple but effective approach to solve the problem and there is no need to develop complicated bi-objective solution approaches to reach equity among drivers when the number of periods is sufficiently large.

These results should provide valuable insights to decision-makers for reaching equity over a given planning horizon for free.

6.2 Limitations

We decided to focus on the CVRP since this is the canonical vehicle routing problem. Other classes of VRPs, which are closer to real-life applications, would however be worth investigating. In Section 6.3 we describe how this limitation in our study may lead to future research directions.

6.3 Future Research

To cover the previously mentioned limitation, further extensions of this work may consider adaptations of our two-phase algorithm for other classes of VRPs. We can think, for example, of VRPs with time windows or even VRPs in stochastic or dynamic settings, which are more realistic and provide opportunities to probe the efficiency of this method for problems that are closer to real life.

Also, we observed that perfect equity is not reached when the number of periods is small (i.e., 2-, 3- and 5-period instances). In this regard, we may investigate an integrated approach where equity is considered when a CVRP is solved in each time period. Furthermore, solving the integrated problem in which routes are constructed while accounting for equity will provide a baseline to compare the performance of our two-phase algorithm.

Finally, another research direction would be to consider the MVRPB with person-oriented consistency. Consistency is a subject that is praised in multi-period contexts and leads to customer and driver satisfaction (Kovacs et al., 2014). There are three different types of consistency: arrival time consistency, delivery consistency, and person-oriented consistency, where the latter encourages drivers to visit as much as possible the same customers over the planning horizon. Clearly, person-oriented consistency is in conflict with cost and equity objectives, since it impacts the choice of customers to be served by each driver in different periods of the planning horizon. Considering person-oriented consistency and equity together in a multi-period setting makes the problem more challenging but closer to real-life applications.

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